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An Investigation into the Role of Quantifiers in Deontic Logics

by

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PREFACE

Four years ago I never would have thought that I would write a thesis in logic but here we are. During my first year at UGent, I followed my first course on logic which was taught by prof. dr. Bert Leuridan and dr. Frederik Van De Putte. I liked this course for a number of reasons. I liked it because it was different from the other courses. It was different in the sense that there were these symbols and ways to manipulate them. There were proofs and goals and reaching such a goal gave a sense of satisfaction.

It was also different in a different sense. It felt like a fresh start. It is not an understatement to say that, coming out of high school, I was not very mathematically oriented nor proficient which, I think, was largely due to not properly knowing the basics and being unmotivated. The fact that this logic course started from the basics and so did not require any previous mathematical knowledge motivated me to keep up with it and do the required work. During this course, I realised that I liked logic and wanted to get a deeper understanding of this discipline which is why after the course was over I reached out to dr. Frederik Van De Putte. I asked him whether he could provide me with any reading tips, which he gladly did, and for which I am thankful.

The following year, luckily, there was another logic course. This course was taught by prof. dr. Joke Meheus. The logic that was taught in this course was more abstract than the previous one which, I discovered, I liked even more. At the end of one of these classes prof. dr. Joke Meheus told a group of students, one of which was me, that she would happily accept students that wanted to complete a Bachelors thesis on a logic oriented topic. This was perhaps the tipping point that led to this thesis. I convinced myself that I wanted to write a Bachelors thesis in logic and so I did under the supervision of prof. dr. Joke Meheus as promotor and dr. Frederik Van De Putte as copromotor.

It felt like the natural next step was to write a masters thesis on the same topic and so it happened. This time I wrote it under the supervision of dr. Frederik Van De Putte as promotor and prof. dr. Joke Meheus as copromotor. The completion of this thesis was not without its hiccups and there were many times that things just didn't go to plan. However, if, in the end, I have produced something that has some value to some people I will be satisfied. I have learned a lot these last four years and there are some people to thank for that. There is, of course, the entire staff that teaches philosophy at UGent of which there are two people in particular that I wish to thank: dr. Frederik Van De Putte and prof. dr. Joke Meheus.

I want to thank my promotor dr. Frederik Van De Putte for teaching me much of what was needed to complete this thesis. If it were not for his critical remarks, corrections and advice this thesis would certainly be of lesser quality. During these last two years of supervision, there were many meetings and each of these meetings often went on for more than an hour but of which every minute was appreciated. I also thank him for stirring up my self-esteem by expressing faith in me that I would complete this thesis when things weren't going my way.

I want to thank my copromotor prof. dr. Joke Meheus for everything that she has taught me during the last three years. If it were not for her open invitation to write a Bachelors thesis under her supervision I might not have ended up writing this thesis. During that Bachelor thesis I have learned much of the basics that were needed to complete this thesis. I also want to thank her for allowing me to take two additional logic courses this year from the *Postgraduate Studies in Logic, History and Philosophy of Science* which have also contributed to a deeper understanding of what philosophical logic is all about.

I want to thank Stef Frijters who collaborated with me this last semester during a course of the aforementioned *Postgraduate Studies in Logic, History and Philosophy of Science* which, as a side-project, gave me some valuable experience on how to approach a philosophical problem by formal means. I also want to thank him for being on the reading committee. Some gratitude is owed to dr. Rafał Urbaniak as well who took the time to listen to my philosophical problems and gave me some feedback when the metaphysics course he taught didn't go as planned.

Lastly, I want to thank my parents for always supporting me in my choice to pursue a philosophy degree and making this a financial possibility. I also wish to thank Lander, Stan and Sam for being splendid room-mates this last year and thanks to Pieter and Stan for proof-reading parts of this thesis.

LIST OF SYMBOLS AND ABBREVIATIONS

Logical symbols

What follows is a list of the most commonly used logical symbols:

$\neg$	Negation
$\supset$	Material implication
$\wedge$	Conjunction
$\vee$	Disjunction
$\equiv$	Equality
$=$	Identity
$\forall$	Universal quantifier
$\exists$	Existential quantifier
$\mathbf{O}$	Deontic necessity
$\mathbf{P}$	Deontic possibility
$\mathcal{S}$	Set of proposition symbols
p, q, r	Proposition symbols
$\mathcal{V}$	Set of variables
x, y, z	Variables
$\mathcal{C}$	Set of individual constants
a, b, c	Individual constants
$\mathcal{P}^r$	Set of predicate symbols
P^r, Q^r	Predicate symbols
A, B, C	Meta-variables for formulas
α, β, γ	Meta-variables for terms
π, π_1	Meta-variables for predicates
$\mathcal{W}$	Set of well-formed formulas
$\in$	Set membership
$\cup$	Union
$\subseteq$	Subset
$\times$	Cartesian product
$\wp$	Power set
W	Set of worlds
R	Accessibility relation
D	Outer-domain
D_w	Inner-domain of w
a	Assignment function
d	Domain function

Abbreviations

iff	If and only if
SDL	Standard Deontic Logic
QFD	Quantified free deontic logic
BF	Barcan formula
CBF	Converse Barcan formula
GF	Ghilardi formula
CGF	Converse Ghilardi formula

SAMENVATTING

In deze masterproef wordt onderzoek gedaan naar de rol van kwantoren in deontische logica's. Kwantoren toevoegen aan een deontische logica is geen rechtlijnig proces. Ik presenteer een variëteit aan mogelijke manieren om dit te doen en bespreek hun formele eigenschappen. Eén van de conclusies van deze presentatie is dat vele van deze deontische predikatenlogica's vrije logica's blijken te zijn. Dit zorgt ervoor dat de manier waarop de deontische operatoren semantisch gedefinieerd zijn herzien moet worden om een meer intuïtieve semantiek te bekomen. Deze revisie zorgt ervoor dat niet bestaande personen geen deontische relevantie meer hebben en wordt de Van Benthem-clausule genoemd. Vervolgens wordt ze expliciet gecontrasteerd met de standaard clausule. Na een reeks formele opties te hebben gepresenteerd ga ik na op welke manier sommige van de formules die interacties uitdrukken tussen deontische operatoren en kwantoren geïnterpreteerd kunnen worden. Ik besluit hieruit dat formules waarbij de kwantoren buiten het bereik van de deontische operator liggen *de re* geïnterpreteerd kunnen worden als uitdrukkingen die iets zeggen over persoonsgebonden normen terwijl formules waarbij de kwantoren binnen het bereik van de deontische operatoren liggen (*de dicto*) een vorm van niet persoonsgebonden normen kunnen uitdrukken. Echter, om deze interpretatie plausibel te maken moeten een aantal axiomakandidaten opgegeven worden, namelijk de Barcanformule, de Ghilardiformule en de converse Ghilardiformule terwijl de converse Barcanformula wordt behouden. Om dit mogelijk te maken hebben we een variërende domeinsemantiek nodig en de adoptie van de Van Benthem-clausule.

CONTENTS

1. INTRODUCTION

"It ought to be the case that everyone is happy", "Someone should rescue that drowning child over there", "No sentient animal should suffer needlessly", "Everyone may drink as much as he or she wants tonight", "Someone is permitted to push that red button over there" all of these expressions have at least two things in common. First, they contain a deontic expression of some sort and second, they contain a quantifier.

Deontic expressions are modal expressions indicative of normative content. The word "deontic" is rooted in the Greek expression "$\delta\acute{\epsilon}o\nu$", which means "what is binding" or "proper" (Hilpinen and McNamara, 2013). Deontic expressions such as "must", "ought", "should", "may", "permissible" and "forbidden" can be found in a wide range of human discourse of which the three most important are probably moral discourse, legal discourse and games. Any kind of area governed by rules of some sort will have among its lexicon deontic expressions of some sort.

Quantifiers are the elements of a sentence indicative of quantity. The meaning of expressions like "everyone" or "someone" are determined by the context in which we are uttering them. They are bound by what we call the *domain of discourse* which is that about which we intend to speak and let our quantifiers range over. If we are at a party and I utter the phrase "It seems like everyone is enjoying themselves" it is understood that by "everyone" I mean everyone at that particular party.

Suppose I said that everyone ought to enjoy themselves at the party I would be asserting a claim that combines a deontic expression with a quantifier. Sentences that combine these two kinds of expressions are what we will call "quantified deontic sentences" of which I have given some examples in the opening paragraph. They are the kind of sentences that are the primary object of study in this thesis and are of importance mainly due to their prevalence and subject matter. To give a sense of their importance we can look at the *Universal Declaration of Human Rights* which contains some examples of quantified deontic sentences.[1] Let us take a look at article 5:

> Article 5: No one shall be subjected to torture or to cruel, inhuman or degrading treatment or punishment. (Assembly, 1948)

What does "no one" refer to exactly? Do we need to evaluate this expression with

[1] Daniel Rönnedal argues in Rönnedal (2014*a*) that we can use quantified deontic logic to analyse the rights contained within the *Universal Declaration of Human rights* which inspired me to use one of the articles as an example at the beginning of this thesis.

reference to all the currently existing people or also to future possible individuals? In other words, what is its intended domain of discourse? Does it follow that it is forbidden for everyone to torture other people? Most people will find no great difficulty in making some correct inferences based on Article 5. However, we shouldn't deceive ourselves into thinking that the meaning of such sentences is always transparent. In reality, we encounter much more opaque sentences or complex constructions of which we can no longer intuitively grasp the subtle nuances and their consequences. To give an example:

1. All of the Frenchmen are wine drinkers.
2. Some of the wine drinkers are gourmets.

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3. Some of the Frenchmen are gourmets.

This is the syllogism presented in an experiment (Oakhill et al., 1989) to 45 subjects all of whom were students who had not taken any classes on formal logic. The majority of these students saw no problem with the conclusion despite the fact that this argument is not valid. These syllogisms were intended to illustrate that if the conclusion is itself believable many people deceive themselves into thinking the argument is valid (Johnson-Laird, 2010). People, generally, are not very good reasoners and our gut-instinct can easily lead us astray. There are, however, many disciplines in which proper reasoning is highly important and, surely, it doesn't need much convincing that ethics and law are two primary examples of this.

This is why it is important to carefully think about what kinds of arguments are valid. Assessing the validity of arguments is perhaps the main objective of logic as a formal discipline. Fundamentally, logic is concerned with the question: what is good reasoning? When we are reasoning about a particular topic, we have to ask ourselves: which inferential steps are valid, which are not, and why is that the case? Formal logic is the attempt to answer these questions in a structured, systematic, and analytical fashion.

Deontic logic is a type of modal logic that has as its subject matter sentences containing deontic expressions. This is nowadays a well-established discipline spanning many decades of research. First-order predicate logic is an extension of propositional logic that is able to deal with quantifiers. This logic is perhaps one of the most established and well-studied logics in existence. If we want to build a logic that can handle deontic expressions with quantifiers, the most straightforward way is to combine a well established deontic logic such as Standard Deontic Logic (**SDL**) with first-order predicate logic. Surprisingly, we quickly discover that combining a modal logic with a predicate logic is not straightforward at all. As Patrick Blackburn and Johan Van Benthem put it:

> We turn now to what is arguably one of the least well behaved modal languages ever proposed: first-order modal logic. [...] Had first-order modal logic never existed, a logician who proposed its (now standard) syntax and relational semantics might have been regarded as audacious, perhaps downright careless. Why? Because, in essence, first-order modal logic is a combined logic. As we have just seen, combining two modal logics while retaining

> interesting properties is no easy matter. So it should not come as too much of a surprise that combining propositional modal logic with first-order logic is unlikely to be plain sailing. (Blackburn and Van Benthem, 2007, p. 66)

What it boils down to is that when constructing such a logic we encounter many formal and philosophical problems that do not occur when we do not take quantifiers or modalities into account. These formal and philosophical problems will not necessarily be a problem of quantified deontic logics in particular but problems of quantified modal logics in general. However, the way in which to resolve these problems or what options are most suitable will, to a large extent, be determined by the fact that we are working with deontic logics. In this thesis, we will take a look at what kinds of philosophical and formal problems these are and how we can deal with them from the deontic point of view.

There are also some important limitations to my discussion of quantified deontic logics. My discussion on the semantical side remains limited to a discussion of standard relational Kripke semantics. There are, however, many different ways in which we can semantically interpret modal predicative languages. Some of these other options will very briefly be touched upon throughout this thesis and I will also later on motivate why I chose to work with Kripke semantics instead of one of these alternatives. My discussion will also remain entirely on the first-order level and I will not look at multi-modal systems combining deontic operators with, for example, alethic or temporal operators.[2] Another important thing to keep in mind is that I will work with the principles of Standard Deontic Logic which is known to be susceptible to many paradoxes. To maintain focus I will have to largely ignore these paradoxes but they always lurk just beneath surface level.

Before we dive into the more formal part of this thesis, I will, in chapter 2, take the time to situate the subject matter of this thesis by providing some historical context. I will also explain how this thesis can be seen as a response to the lack of attention quantified deontic logic has received so far. Normally, at the start of a thesis, there is a chapter dedicated to the *status quaestionis* of the subject matter, however, because of the lack of attention and the resulting lack of systematic work on the topic there is hardly enough content to produce a coherent chapter. Of course, the relevant literature will be weaved in throughout the entirety of this document. The last section of chapter 2 will be used to reflect on what it is that logicians actually do.

This is a thesis in philosophy and more specifically its domain is that of philosophical logic. This is why one of the aims of chapter 3 is simply to show what a formal logic that can be used for philosophical analysis looks like. We will see that a formal logic consists of three main constituents: a formal language, a syntax and a semantics. Each of these constituents have to be designed to suit the specifics of a first-order deontic logic. This prompts a lot of questions. Which options are there when it comes to the syntax of a first-order deontic logic and what are the important formulas that express interactions between quantifiers and deontic operators? What does a model that is able to interpret a first-order deontic

[2] If one is interested in some of the technicalities of these kinds of systems see Rönnedal (2014*b*).

language look like and what are the formal options with respect to standard relational Kripke semantics? How does the semantics relate to the syntax of a logic? In short: the main aim of this chapter is to show the plethora of formal options that the deontic logician faces when bringing quantifiers into play.

In chapter 4, we will inquire into the meaning of formulas that express relations between the deontic operators and the quantifiers. We will do so by examining what it is that these formulas semantically express by using the various kinds of models introduced in chapter 3. The main aim of this chapter is to use the semantic meaning of these formulas to advance an interpretation of them which will help us make some choices with respect to the formal options presented in chapter 3.

In chapter 5, I will synthesise what has been said so far and the conclusions that we can draw from this. I will highlight some of its shortcomings and limitations and share some of the options for future research that I have stumbled upon when surveying the literature.

2. WHAT, WHY AND HOW?

In this chapter I will situate the topic in its historical context, explain why we should tackle the problems that it presents and how to do that.

2.1 A very short history of deontic logic

2.1.1 The dawn of modality

Deontic expressions fall under the umbrella of modal expressions. Modal expressions are a key component of every language and yet it proves elusive to define exactly what is meant by the term "modality". Clarifying its meaning usually consists in giving some examples. They are expressions like: necessarily, possibly, it is obligatory that, it is permissible that, it ought to be that, it has always been that, it will always be that, etc. The common thread throughout all these modal utterances is that they speak not merely of the actual, of what is, but of ways in which the world might have been, ought to be, will be etc. (Melia, 2014).

It is clear that these kinds of expressions pervade our everyday language[1] and hence our everyday thinking. Alan R. White (White, 1975) suggests that modality lies at the root of many of our oldest philosophical problems. He writes: "...scepticism is based on the feeling that nothing can be known unless the possibility of its being otherwise has been ruled out; while determinism usually enshrines the belief that it is not possible for anything to be otherwise than it is. The problem of free will is the problem whether anyone could have done something other than what he did do..." and concludes with "Of many queer philosophical views it is true to say that there is modality in their madness." (White, 1975, p. 2).

Because modality is so important in both everyday and philosophical thinking, it is no surprise that philosophers have been concerned with the meaning of modal statements as early as antiquity. Aristotle is one of the first philosophers who concerned himself with modality in a philosophical way when he busied himself with modal syllogisms in his work "*Prior Analytics*" (Smith, 1989).

In this thesis, the focus will be on deontic modalities: modalities like "ought to be", "ought to do", "it is obligatory that", "it is permissible that", etc. The first logical investigations into normative modalities took place in the fourteenth century, nearly a thousand years after Aristotle. According to Knuuttila (1981), the

[1] Henceforth I will use the term "natural language" to refer to everyday language.

first discussion of a logic of norms in which the deontic notions are treated analogously with the interdependencies of alethic modal notions like "necessity" and "possibility" is to be found in the work of Roger Rosetus, of whom almost nothing is known, and Robert Holcot, an English Dominican scholastic philosopher.

2.1.2 Calculemus!

Fast forward another 300 years to the seventeenth century and we find Gottfried Wilhelm Leibniz discussing deontic concepts such as the obligatory (*debitum*), the permitted (*licitum*) and the prohibited (*illicitum*) in his work *Elementa iuris naturalis* and calls them "modalities of law" (*iuris modalia*) (Hilpinen and McNamara, 2013; Leibniz, 1930). He also establishes a connection between alethic modal logic and ties it to legal modalities.

It is also Leibniz who, in his dream, gave birth to the concept of a *calculus ratiocinator*, which would reduce any dispute to a mere calculation:

> quando orientur controversiae, non magis disputatione opus erit inter duos philosophus, quam inter duos computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: calculemus (Leibniz and Gerhardt, 1875)
>
> When controversies arise, there will be no more need for a disputation between two philosophers than there would be between two accountants [computistas]. It would be enough for them to pick up their pens and sit at their abacuses, and say to each other (perhaps having summoned a mutual friend): 'Let us calculate.' (Ross, 1984)

It is believed by some (e.g. Fearnley-Sander (1982)) that Leibniz's idea of a *calculus ratiocinator* anticipates mathematical logic. In some ways, provided a charitable reading, he even anticipated deontic logic when he wrote:

> ...moral ideas are more complex than the figures ordinarily considered in mathematics, and that makes it hard for the mind to retain the precise combinations of constituents of moral ideas as perfectly as is needed for long deductions. If in arithmetic the various stages weren't indicated by marks whose precise meanings are known and which last and remain in view, it would be almost impossible to perform long calculations. In moral discourse definitions provide some remedy for this trouble; provided they are kept to. And what methods algebra or something like it may some day suggest to remove the other difficulties - who can tell? (von Leibniz et al., 1996, p. 385)

Whether or not Leibniz can be said to have anticipated it, the spirit of a *characteristica universalis*, a universal and formal language that was to empower the *calculus ratiocinator* envisioned by him, is somewhat exemplified in the work of Gottlob Frege who first introduced quantifiers and predicates in his *Begriffsschrift* (Isaac, 2016). Although Frege himself notes that Leibniz was a tad too optimistic concerning a calculus ratiocinator: "Auch Leibniz hat die Vortheile einer angemessenen Bezeichnungsweise erkannt, vielleicht überschätzt. Sein Gedanke einer allgemeinen Charakteristik, eines calculus philosophicus oder ratiocinator war zu riesenhaft, als dass der Versuch ihn zu verwirklichen über die blossen Vorbereitungen hätte hinausgelangen können."[2](Frege, 1879*b*, p. 11).

2.1.3 The build up to formal deontic logic

The work of Frege was not isolated but an extension of the work already done on propositional logic by logicians such as George Boole (Boole, 1854), Augustus De Morgan (De Morgan, 1847), William Stanley Jevons (Jevons, 1864, 1872), John Venn (Venn, 1881) and others (for a detailed history of formal logic see Bochenski and Thomas (1970)).

All of this earlier work laid the necessary groundwork for formalised modal logic. The publication of C.I. Lewis's *Survey of Symbolic Logic* (Lewis, 1918) is considered by Blackburn et al. (2002) as the birth of modal logic as a mathematical discipline. According to Blackburn et al. (2002) "Lewis's work sparked interest in the idea of 'modalizing' propositional logic, and there were many attempts to axiomatise such concepts as obligation, belief and knowledge (Blackburn et al., 2002, p. 38-39).

This brings us to the first "proper" formalizations of deontic logic. Among them was the system named "Deontik" by the Austrian philosopher Ernest Mally (Mally, 1971). What really took deontic logic of the ground, however, was the seminal paper by George Henrik von Wright matter-of-factly named "Deontic logic" (Von Wright, 1951). Woleński (1990) brings attention to the fact that similar ideas were independently developed in Poland by Jerzy Kalinowksi (Kalinowski, 1953) and in Germany by Oskar Becker (Becker, 1952).

All of this work culminated in what would be known as Standard Deontic Logic. Not standard in the sense that it is widely accepted, but standard in the sense that it is used as a point of reference. That it is widely considered only as a starting point has to do with its many paradoxes (see Hilpinen and McNamara (2013)).

2.1.4 A call to arms!

What is striking though is that almost all work in deontic logic in the timespan between the publication of Wright's "Deontic Logic" and the present has been done on the propositional level. This was true in 1983 when Jan Štěpán wrote:

> "Systémy normativní logiky vybudované na bázi výrokové logiky lze dnes považovat již za klasické. Problém vytvoření normativní logiky na základě predikátové logiky prvního řádu dosud nebyl systematicky zkoumán."(Štěpán, 1983, p. 67)

[2] Translation: "Leibniz, too, recognized -and perhaps overrated- the advantages of an adequate system of notation. His idea of a universal characterisic, of a *calculus philosophicus* or *ratiocinator*, was so gigantic that the attempt to realize it could not go beyond the bare preliminaries." (Frege, 1879*a*, p. 6)

Which translates to "Normative logic systems built on the basis of propositional logic can now already be considered standard. The problem of creating a normative logic based on deontic first-order logic has not been systematically studied.".

This fact unfortunately remains true for the years that came after Štěpán's observation. Illustrative of this state of affairs are the last two international ΔEON-conferences. In both the 11th (*Deontic Logic in Computer Science*) as well as the 12th ΔEON-conference (*Deontic Logic and Normative Systems*) none of the 33 articles presented were about first-order deontic logic (Cariani et al., 2014; Agotnes et al., 2012).

Another example is the "*Handbook of Deontic Logic and Normative Systems*". In it we read: "This handbook presents a detailed overview of the main lines of research on contemporary deontic logic and related topics." (Gabbay et al., 2013, p. vii). However, the entire book contains only a handful of pages about deontic logic at the predicative level and Risto Hilpinen and Paul McNamara remark "Most presentations of deontic logic are restricted to propositional logic. This is a serious and unnecessary limitation; [...] some normative propositions and relations can be formalized in a plausible way by combining deontic operators and quantifiers.(Hilpinen and McNamara, 2013, p. 51).

This is a quite unexpected state of affairs given the nature of our deontic discourse which contains an abundance of sentences combining quantifiers and deontic expressions. This is especially surprising because Jaakko Hintikka, a Finnish logician, made a call to arms to equip deontic logic with quantifiers as early as 1957 (Hintikka, 1957) when he wrote:

> ...quantifiers seem to me indispensable for any satisfactory analysis of the notions with which every system of deontic logic is likely to be concerned. Among these notions, perhaps the most important ones those of obligation, forbiddance, permission and commitment. I shall argue that, in the contexts contemplated by the deontic logicians, the logical relations of these notions cannot be described without using such words as 'every' and 'some'. (Hintikka, 1957, p. 4)

We can only guess as to why this is the case but some suggest that it is due to the inherent complexities of the subject. As Patrice Bailhache puts it in his *Essai de logique déontique*:

> Il nous a fallu de même résolument ignorer le calcul des prédicats. Et cependant, avec l'introduction des individus dans les modalités, sa mise en oeuvre était fortement suggérée. Mais, si nous nous étions engagés dans cette voi, il nous aurait fallu traiter des questions d'existence en relation avec les modalités, et ces questions difficiles, bien que fort importantes philosophiquement, nous auraient entraîné dans des développements d'une effroyable complexité.[3] (Bailhache, 1991, p. 9)

[3] Own translation: "We also had to resolutely ignore the predicate calculus. And yet, with the introduction of individuals into the modalities, its implementation was strongly suggested. But if we had taken this path, we would have had to deal with questions of existence in relation to the modalities, and these difficult questions, though very important philosophically, would have

Given this apparent complexity, why should we take the trouble? This is the question that I hope to answer in the following section.

2.2 Why should we care?

Why should we care about applying logic to our normative reasoning and why should we extend propositional deontic logic to include quantifiers and predicates? First I will motivate why it is useful to investigate our normative thinking from a logical point of view and thereafter I hope to convince the reader that it is worthwhile to investigate the predicative case.

One often finds among laymen the conception that logic and ethics are two distinct or even opposing realms of human inquiry; the one cold and objective, the other humane and subjective. Not so, argues John Corcoran in his article *The inseparability of logic and ethics*. I agree with him when he concludes his article with the following words:

> I have in mind the fact that logic can be seen as an ongoing, imperfect, incomplete, and essentially incompletable attempt to cultivate objectivity, to discover principles and methods that contribute to the understanding and practice of objectivity, which is an ethical virtue standing alongside kindness, justice, honesty, compassion, and the rest, and which is characteristically human in the sense that an omniscient or infallible entity would have no use for objectivity and no use for logic. Logic is a humane and humanistic science; it is one of the humanities in the renaissance sense. (Corcoran, 1989, p. 40)

In order to cultivate this objectivity, we have to abstract away from the particularities of natural language and make use of a formal language. Such a logical formalisation can serve at least three functions. It has a descriptive function because we are able to describe in precise terms the many ways in which humans reason by laying bare the premises and inferential steps of the arguments (Batens et al., 2009). It also has an explanatory function: it allows us to explain how someone arrived at a conclusion and if the conclusion does not follow from the premises explain why. Lastly, it has a prescriptive function, because we can diagnose problems in the arguments, show precisely where they went awry and as such prescribe how to reason correctly.

Harry Gensler puts it as follows:

> Logic can help us understand our moral reasoning - how we go from premises to a conclusion. It can force us to clarify and spell out our presuppositions, to understand conflicting points of view, and to identify weak points in our reasoning. Logic is a useful discipline to sharpen our ethical thinking. (Gensler, 1996, p. 35)

Let us now consider the second part of my motivation. Why should we care about adding quantifiers and predicates to our formal language and hence complicate

led us into frighteningly complex developments."

matters? The reason is quite straightforward: our everyday language contains many uses of quantified deontic sentences and so if we are able to deal with them we extend the reach of formal logic considerably. Hintikka and Štěpán are not the only ones to notice this need for extending deontic logic.

In the legal sphere there was Ulfrid Neumann, who wrote in 1989 that standard propositional logic was inadequate for the formalisation of "Rechtsnormen und Rechtssätzen":

> Unabhängig von diesem Problem der Darstellbarkeit inhaltlicher Verknüpfungen ist der Aussagenkalkül wegen seiner beschränkten Ausdrucksmöglichkeit für die Formalisierung von Rechtsnormen und Rechtssätzen wenig geeignet; der Unterschied zwischen generellen und singulären Sätzen, der für die Struktur der juristischen Subsumtion wesentlich ist, kann im Aussagenkalkül nicht erfasst werden. Für die Formalisierung von Rechtsnormen und -sätzen wird deshalb überwiegend auf den Prädikantenkalkül zurückgigriffen...[4] (Büllesbach et al., 1989, p. 261)

Someone who has explicitly argued for the use of quantified deontic logic across several articles is Daniel Rönnedal. As noted in the introduction, Rönnedal (2014*a*) argues that we can use quantified deontic logic to analyse the rights contained within the *Universal Declaration of Human Rights*. In Rönnedal (2015*c*) he argues that some normative standards seem to be tied to certain geographical areas and that we can model these intuitions with the help of a quantified deontic logic. In Rönnedal (2015*a*) he takes a more broad look at universal norms and the structure of normative systems. And lastly, he argues for a quantified deontic logic within the context of an analysis of the free choice permission paradox (Rönnedal, 2015*b*). Unfortunately some of these articles are not accessible due to a language barrier.

And so, reassured that there seems to be a need for more reflection on quantified deontic logic I will do so in the following chapters of this thesis. But first I will reflect on how to proceed and what it is that we need before we dive more deeply into the subject matter.

2.3 Methodological reflections

Before reflecting on how to achieve our goal we first need to know what it is. One of the goals of a deontic logician is to model deontic reasoning in a formally precise way. In this thesis we use a first-order modal language which will give us

[4] Own translation: "Irrespective of this problem of the representability of content connections, the calculus of propositions is not very suitable for the formalization of legal norms and sentences because of its limited expressiveness; the difference between general and singular sentences, which is essential for the structure of legal subsumption, can not be expressed within a proposition calculus. Thus, the formalization of legal norms and sentences will largely be based on the calculus of predicates."

more expressive resources. One of the goals thus consists in discovering whether and how we can use these extra expressive resources to model our deontic reasoning in a better and more precise way than was previously possible. As we have seen, such a formalisation can serve at least three functions. It is important to keep these functions in mind when assessing the adequacy of a logic. They are essential in determining the qualities we should strive for in a logic. The way in which logicians choose between competing "theories" is comparable to the empirical sciences. They produce criteria that allows them to compare and assess the strengths and weaknesses of various logical systems and thus allow them to pick one over the other.

These are criteria like parsimony, fruitfulness and adequacy to the data. A logic shouldn't be needlessly complex: we need to be parsimonious with regards to its internal structure and not add unnecessary components. Nevertheless, we should also aim at fruitfulness, a logic shouldn't be idiosyncratic to the point of being applicable to only a very small subset of human reasoning. This is especially the case here because first-order deontic logic is relatively unexplored and so we should try to preserve options wherever possible. It should also show adequacy to the data, a theory that is detached from reality can not give a proper account of human reasoning nor have any normative import.

But what constitutes data in logic? Our data will have to be samples of human reasoning. This is the kind of data that allows us to accomplish the descriptive and normative function. We look at specific cases of reasoning to determine how we reason and, to assure us of normative import, search for examples of reasoning that we are convinced are sound and of which we can easily convince others that they should serve as a benchmark of good reasoning. Any logic should then be able to account for them and if it cannot, it is agreed among the people that accepted their soundness that this reveals a weakness of the logic.

The fact that logics can serve multiple functions and are tailored to specific domains of human reasoning almost necessarily entails a logical pluralism. It is quite implausible that it is possible to construct a logic that models the whole range of human reasoning while still complying with the criteria of parsimony, fruitfulness and adequacy to the data. I will not advance additional arguments for this plurality view but it is implicitly assumed in this thesis.

In the chapter ahead I will show what some of our options are with respect to constructing a quantified deontic logic.

3. QUANTIFIED DEONTIC LOGIC

Logic is concerned with the question: what is good reasoning? However, as previously indicated, we will only take a look at a small fragment of human reasoning. They way in which this fragment is formalised will be the subject of this chapter. Before I spell out the formal machinery needed to study quantified deontic logic in particular, it is useful to get a sense of how a logic is constructed in general.

A logic typically consists of three parts: a formal language, a syntactic part, and a semantic part. A formal language is a language made up of precisely picked symbols and the rules by which to combine them into formulas. Naturally, we have to pick our symbols and our rules for combining them in such a way as to be able to express the relevant aspects of the kind of arguments under consideration.

In the syntax of our logic, we concern ourselves with the way in which formulas are related to each other. One way of achieving this goal is by use of an axiomatic system. In an axiomatic system, we specify the rules by which to derive formulas from each other and pick the formulas that will function as the axioms of our system. The formulas that we are able to derive from these axioms are its theorems. In the syntactic part, we do not concern ourselves with meaning. In the words of Ernest Nagel: "The postulate and theorems of a completely formalized system are "strings" of *meaningless marks*, constructed according to rules for combining the elementary signs of the system into larger wholes."(Nagel et al., 2001, p. 26). If the symbols can be said to have meaning it is only in the way that they relate to each other as a consequence of their behaviour within the system and not as a consequence of something external that imbues them with meaning. Why do we want such a highly formalised system? Because, as Nagel notes, "...it serves a valuable purpose. It reveals structure and function in naked clarity."(Nagel et al., 2001, p. 26).

In the semantic part of our logic, we concern ourselves with the way in which formulas are related to their possible referents (Tarski, 1969). That is, with the way in which the symbols and formulas relate to things outside the formal language. We study this by using models. A model is a mathematical structure that interprets a formal language. By an interpretation of the formal language is meant a fixing of the referent of every formula and non-logical symbol in the formal language to only one of its possible referents. In other words, each particular model maps the non-logical symbols of the formal language and, consequently, its well-formed formulas onto a possible referent in accordance with the rules of the structure. By using a model-theoretic apparatus external to the language itself, logicians are able to discover interesting results about formal languages, their semantic properties, and the relation between syntactical and semantic features of a logic (Chang and Keisler, 1990).

In this chapter, I will firstly construct a language that enables me to discuss the philosophical problems that arise in discourse containing quantification and deontic modalities. Secondly, I will expound on how the syntax of a quantified deontic logic may be given shape. Thirdly, I will elaborate on possible ways to construct model-theoretic interpretations of a formal language adequate for expressing first-order logic with deontic modalities. Lastly, I will clarify some aspects of the relation between syntax and semantics.

3.1 The language

In this section, I will explicate the non-logical symbols and the logical symbols that I am going to use throughout this thesis and how to construct well-formed formulas from them.

Firstly, we need symbols that represent ordinary propositions. I will use the set $\mathcal{S} = \{p, q, r, s, t, u, p_1, ...\}$ and call them *proposition symbols*. Secondly, we need symbols that range over individuals without targeting someone specifically, I use the set $\mathcal{V} = \{x, y, z, x_1, ...\}$ and call them *variables*. Thirdly, I will use the symbols of the set $\mathcal{C} = \{a, b, c, d, e, a_1, ...\}$ to refer to specific individuals and call them *constants*. Members of the set $\mathcal{C} \cup \mathcal{V}$ are called *terms*. Lastly, the set $\mathcal{P}^r = \{P^r, Q^r, T^r, P^r_1, ...\}$ are the symbols used as *predicates*, the rank r displays the arity of the predicate.[1]

In addition to the non-logical symbols, we need some logical symbols. They are the symbols of the set $\{\neg, \supset, \wedge, \vee, \equiv, =, \forall, \exists, \mathbf{O}, \mathbf{P}\}$ and are called, in order, negation, material implication, conjunction, disjunction, equality, identity, universal quantifier, existential quantifier, deontic necessity, and deontic possibility. $\mathbf{O}$ is taken as primitive and $\mathbf{P}$ is defined as $\neg\mathbf{O}\neg$. Likewise, $\forall$ is primitive and $\exists$ is defined as $\neg\forall\neg$. $\mathbf{O}$ and $\mathbf{P}$ are the deontic operators and their particular interpretation will depend on the context, for example, $\mathbf{O}$ will sometimes be rendered as "it is obligatory that" and sometimes as "it ought to be that".

Furthermore, I will use some meta-variables. The symbols of the set $\{A, B, C, ...\}$ are used as variables for strings of symbols. The members of the set $\{\alpha, \beta, \gamma, ..., \}$ are used as variables for members of the set $\mathcal{C} \cup \mathcal{V}$ and the members of $\{\pi, \pi_1, ...\}$ are used as variables for the members of $\mathcal{P}^r$.

The set $\mathcal{W}$ is the minimal set that satisfies the seven conditions below:

1. $\mathcal{S} \subset \mathcal{W}$.
2. If A $\in \mathcal{W}$, then $\neg$ A $\in \mathcal{W}$.
3. If A $\in \mathcal{W}$, then $\mathbf{O}A$ and $\mathbf{P}A \in \mathcal{W}$.

[1] Whenever the arity of a predicate is unimportant I will not mention the rank r.

4. If A, B $\in \mathcal{W}$, then $(A \vee B)$, $(A \wedge B)$, $(A \supset B)$, $(A \equiv B) \in \mathcal{W}$.
5. If $\pi \in \mathcal{P}^r$ has rank R and $\alpha_1\alpha_2...\alpha_r \in \mathcal{C} \cup \mathcal{V}$, then $\pi\alpha_1\alpha_2...\alpha_r \in \mathcal{W}$.
6. If α, $\beta \in \mathcal{C} \cup \mathcal{V}$, then $\alpha = \beta \in \mathcal{W}$.
7. If A $\in \mathcal{W}$ and $\alpha \in \mathcal{V}$, then $(\forall\alpha)$A, $(\exists\alpha)$A $\in \mathcal{W}$.

Each member of $\mathcal{W}$ is a well-formed formula of the language, in other words, $\mathcal{W}$ is the set of all well-formed formulas. Brackets are used to disambiguate wherever needed. The usual conventions with respect to quantifiers are adopted:

Definition 1. *The scope of a quantifier is the formula that immediately follows it.*

Definition 2. $\alpha \in \mathcal{V}$ *is a free variable iff* α *does not occur within the scope of a quantifier over* α.

Definition 3. $\alpha \in \mathcal{V}$ *is bound by a quantifier over* α *iff* α *occurs free in the scope of that quantifier over* α.

Definition 4. *A formula* A *is closed iff no member of* $\mathcal{V}$ *occurs free in* A.

Definition 5. *A formula* A *is open iff a member of* $\mathcal{V}$ *occurs free in* A.

Definition 6. $A[\alpha]$ *means that the formula A has free occurrences of* $\alpha \in \mathcal{C} \cup \mathcal{V}$.

Definition 7. $A[\beta/\alpha]$ *is the formula obtained by substituting every free occurrence of* $\alpha \in \mathcal{C} \cup \mathcal{V}$ *in* A *by* $\beta \in \mathcal{C} \cup \mathcal{V}$ *on the condition that* β *does not occur free in* A.

3.2 The semantics

Models are mathematical structures used to interpret a formal language. Generally, the formulas that are satisfied in a model are the formulas said to be true under its interpretation. In modal logic, formulas are satisfied relative to a point of evaluation in the model. The truth of a formula is thus also contingent on the point of evaluation in the model. These points of evaluation are often called possible worlds.

The idea of thinking about modality in terms of possible worlds is certainly not new: it can be traced back to as early as the 18^{th} century when Leibniz declared the actual world to be the best of all possible worlds (Leibniz and de Jaucourt, 1747). Saul Kripke is one of the first logicians who took this idea and forged it into a formal structure (Kripke, 1959, 1963*b*). However, there is still some dispute about who deserves credit for what. Jaakko Hintikka, Stig Kanger, Arthur Prior and Richard Montague are all logicians who have, in one way or another, contributed to this formalization and deserve some part of the credit (Blackburn et al., 2002; Goldblatt, 2006).

There are usually a variety of ways we can construct models. In the case of quantified modal logics this is especially true and the choices we make will be a reflection of our philosophical choices.

Traditionally, Kripke-semantics for *propositional* modal logic uses a model that is a triple $\langle W, R, v\rangle$ consisting of a Kripke-frame $\langle W, R\rangle$ and a valuation function v. W is a set of worlds, R is a binary relation on W such that $R \subseteq W \times W$. We call R the accessibility relation: it determines which worlds are accessible from a world. The accessible worlds are the worlds that are relevant to determine the truth of a formula containing modal operators. The function v assigns truth values to the formulas at each w in W.

There are, however, other ways to characterise the semantics of a quantified modal logic such as neighbourhood semantics (also known as Scott-Montague semantics) or David Lewis's counterpart semantics (Lewis, 1968). As indicated, I will only take a deeper look at traditional Kripke semantics. To motivate this choice it is useful to briefly look at how these other kinds of semantics differ from traditional Kripke-semantics. I will only explicate the relevant parts of these models but, for the interested reader, I will also provide some references to the relevant literature if one wants to get a deeper sense of how these semantics work.

Neighbourhood semantics for *propositional* modal logic uses a model that is a triple $\langle W, \mathcal{N}, v\rangle$ consisting of a *neighbourhood-frame* $\langle W, \mathcal{N}\rangle$ and a valuation function v. W is a set of worlds, $\mathcal{N}$ is a function $\mathcal{N} : W \to \wp(\wp(W))$ that maps worlds to sets of sets of worlds. The valuation function v is defined as $v : \mathcal{S} \to \wp(W)$. To see how truth in a model of a formula at a world is defined I refer the reader to the PhD thesis of Kirsten Segerberg (Segerberg and Universitet, 1971). (Waagbø, 1992) has shown how to extend neighbourhood semantics to the predicative level and (Stolpe, 2003) has expanded on this.

Lewis developed counterpart semantics because he saw a problem with identification across possible worlds, i.e. the problem of *transworld identity* (for discussion on this see (Loux, 1979)). When, for example, one says that it is obligatory that Peter does the dishes, then in Kripke semantics Peter will do the dishes in each accessible world.[2] This, however, requires that we can identify Peter across possible worlds contends Lewis and this is seen as problematic. His solution is to introduce a *counterpart relation*:

> The counterpart relation is our substitute for identity between things in different worlds. Where some would say that you are in several worlds, in which you have somewhat different properties and somewhat different things happen to you, I prefer to say that you are in the actual world and no other, but you have counterparts in several other worlds (Lewis, 1968, p. 114)

Formally, a *counterpart-frame* will be a quintuple $\langle W, R, D, d, \mathcal{C}\rangle$. W and R are as before. Because counterpart semantics is a type of semantics developed for the predicative level, we also have an outer-domain D that can be thought of as a set

[2] How this works precisely will become clear in the sections ahead but this should give one a feeling for Lewis's problem.

of individuals, and a function d defined as $d : W \to \wp(D)$. d associates with each world w an inner-domain, i.e. the set of individuals existing at each w denoted with the set D_w. $\mathcal{C}$ can then be defined as a function that assigns a subset of $D_w \times D_{w'}$ to every couple $\langle w, w' \rangle \in R$. This should suffice to give an idea of how this approach would formally look like. To see how a complete model and truth would be defined in these models, see (Braüner and Ghilardi, 2007; Corsi, 2002*a*; Belardinelli, 2006).

There are two reasons why I will use traditional Kripke semantics. The first is that it is the most used semantics and so my analysis will fit in more neatly with the rest of the literature on quantified modal logic. The second reason is that in comparison with neighbourhood semantics and counterpart semantics we can more intuitively interpret the mathematical structure. Thinking in terms of sets of sets of worlds is more opaque than thinking in terms of accessible worlds and, similarly, adding another function or an additional relation on the structure complicates the picture even more. Of course, there are also time constraints at work here. Given enough time, it might prove insightful if we contrasted kripke-models with these other kinds of models. That, however, is left for another time.

3.2.1 Models for quantified deontic logic

If we want models that are able to give interpretations to a language with quantifiers and modal operators we need a more complex structure than the Kripke models briefly introduced in the previous subsection. The most general way to characterize a model of quantified deontic logic that uses Kripke style possible world semantics is one with varying domains.

Definition 8. *A varying domain model is a quintuple $\mathcal{M} = \langle W, R, D, d, a \rangle$. W is a non-empty set of points. $R \subseteq W \times W$ is a serial relation on W. D is a non-empty set. d is a function: $d : W \to \wp(D)$ that associates with each point $w \in W$ a set D_w such that $D_w \subseteq D$. a is a function that satisfies the following conditions:*

1. $\mathcal{C} \cup \mathcal{V} \to D$
2. $\mathcal{P}^r \times W \to \wp(D^r)$
3. $\mathcal{S} \times W \to \{0, 1\}$

Given a model $\mathcal{M}$ the valuation of $\mathcal{M}$ works as follows:

1. $\mathcal{M}, w \vDash A$ *if* $A \in \mathcal{S}$ *and* $a(A, w) = 1$
2. $\mathcal{M}, w \vDash \neg A$ *iff* $\mathcal{M}, w \nvDash A$
3. $\mathcal{M}, w \vDash A \vee B$ *iff* $\mathcal{M}, w \vDash A$ *or* $\mathcal{M}, w \vDash B$
4. $\mathcal{M}, w \vDash \pi\alpha_1...\alpha_n$ *iff* $\langle a(\alpha_1), ..., a(\alpha_n) \rangle \in a(\pi, w)$
5. $\mathcal{M}, w \vDash \alpha = \beta$ *iff* $a(\alpha) = a(\beta)$
6. $\mathcal{M}, w \vDash \mathbf{O}A$ *iff* $\mathcal{M}, w' \vDash A$ *for all* w' *for which* Rww'

7. $\mathcal{M}, w \vDash (\forall\alpha)A$ *iff for all* $\mathcal{M}' = \langle W, R, D, d, a' \rangle$*, where* a' *differs at most from* a *concerning the value it assigns to* α *from* D_w*,* $\mathcal{M}', w \vDash A$ *holds.*

The truth values of all formulas of the language are determined by a valuation of the model $\mathcal{M}$. Each valuation determines the truth of a formula relative to a model and a world. Clause 6 of the valuation will be referred to as the *standard deontic operator clause* because it will, later on in this document, be contrasted with another way in which we can define this clause. The following definitions define truth at a world in a model and validity of a formula:

Definition 9. A *is true at* w *in the model* $\mathcal{M}$ *iff* $M, w \vDash A$*.*

Definition 10. A *is valid iff for every model* $\mathcal{M}$*,* $\mathcal{M}, w \vDash A$ *holds.*

The points in W are usually thought of as possible worlds. But what exactly are "possible worlds"? To answer this question we first need to disambiguate what is meant by "possible". The meaning of "possible" is certainly not fixed. It could mean that which is conceivable. It could mean that which is possible given the laws of nature or perhaps what could be the case given our current technological capabilities. And so, what the relevant possibilities are will differ from context to context and needs to be specified in advance.

It is conceptually useful to introduce a distinction between conceivable worlds and possible worlds. I will use the term "conceivable world" for the worlds we are able to imagine and make intuitively sense. I take it that conceivability is not dependent upon a point of view and that everyone, in principle, could imagine the same state of affairs. Each point in W should be thought of as such a conceivable world. I will use the elements of the set $\{w, w', w'', ...\}$ as variables for conceivable worlds and the elements of the set $\{w_1, w_2, ...\}$ as names for conceivable worlds. The term "possible world", however, will be used as a term relative to other worlds. To determine this relativeness we need a relation R over the set W.

The relation R in the model is called, like before, the *accessibility relation*. For each world, the accessibility relation selects among the conceivable worlds those world which are alethically acceptable from the point of view of that world given the intended meaning of possibility. The alethically acceptable worlds for a world are what we will call the possible worlds for that world. The possible worlds for a given world will thus always be a subset of the conceivable worlds. Moreover, in deontic logic, the accessibility relation picks for each world among the set of possible worlds of that world those worlds which are deontically acceptable[3] for

[3] In McNamara (1996) Paul McNamara defends the view that we should opt for speaking in terms of acceptable worlds and not in terms of deontically optimal or ideal worlds because this conflates the notion of deontic necessity and "ought". An action is deontically necessary if it is required by morality's demands, but surely, McNamara contends "...there is little pre-theoretic support for the contention that morally ideal behavior is always morally mandatory. Does the moral exemplar not at once often do what is morally ideal and what is morally optional?" (McNamara, 1996, p. 164). McNamara is not the first to criticize the use of deontic perfect

that world. A possible world w' is deontically acceptable for a world w if at w' everyone adheres to the rules at w without exception. The relation R is serial which means that for every world w there is a world w' such that Rww'. In summary, the accessibility relation picks for each world w a set of possible worlds from the set of conceivable worlds that are alethically acceptable and then selects a subset of this set of possible worlds in which everyone adheres to the rules at w.

D, which we will call the *outer domain*, should be thought of as the set containing the conceivable individuals. They are the individuals of which it makes intuitively sense to talk about. We can, for example, say intelligible things about Santa Claus, about Socrates or about Michelle Obama. The domain function d associates with each conceivable world the individuals that actually exist at that world. We use D_w to refer to the domain of a world w which consists of the individuals that exist at w. We call it the *inner domain*. It needn't be the case that every conceivable individual actually exists at a conceivable world. Santa Claus might not exist in any conceivable world, while Socrates will exist in some conceivable worlds and Michelle Obama happens to exist in the conceivable worlds which are like our world at present.

However, the entities in the domain of the conceivable worlds needn't be restricted to individuals. They can be arbitrary objects which would make it possible to interpret a sentence such as: "Everyone has to bring some present to the party" where "some" is interpreted as an existential quantifier. However, for reasons of simplicity I will stipulate that the domains are made up of individuals only. Historically, individuals were not always the predominant option. Hintikka took another approach in his 1957 paper "Quantifiers in deontic logic" (Hintikka, 1957). He let the quantifiers range over so called *act-individuals* which is a term that originates with Von Wright (1951). To give an idea of how this term is used: if theft is an act, every instance of thieving is considered an act-individual. Hintikka, for example, uses the formula $(\forall x)(Px \supset \mathbf{O}(\exists y)Fxy)$ to express "any particular act of promising implies an obligation to do at least one act which fulfils the promise in question. If 'Pa' means that the act a is an instance of promising and if 'Fab' means that the act b fulfils the promise given in a." (Hintikka, 1957, p. 18). While this approach has some appeal, there have been convincing arguments by David Makinson in his paper "*Quantificational Reefs in Deontic Waters*" that act-individuals are not the appropriate universe of discourse. He gives the following example:

> Imagine that a few days later you are in the waiting room of another, more easy-going, hospital. You ask a passing nurse whether it is forbidden to smoke. "No", she replies, "it isn't forbidden, though we don't actually encourage it." Hintikka, who is with you again, interprets: "She means", he confides, "that it is not the case that for every act x, it is forbidden that x be a case of smoking here." As you know at least a little classical logic, you reflect: "Aha. So there are some acts x, for which it is not forbidden that they be cases of smoking in here - in other words, $(\exists x)\neg\mathbf{F}Sx$." Then, carried away

worlds. For more on this debate I refer the reader to Hansson (2006) and Purtill (1973). In this thesis, I will, for this reason, speak in terms of deontically acceptable worlds.

> by your reflections, you suddenly ask, "But which ones are they?" My present act of playing with worry beads - is it such an x? That fellow's chewing on gum, right now - is it one of the x's of which it is not forbidden that they be acts of smoking? And that actual case of smoking over there - does it provide an instance of our existential?" (Makinson, 1981, pp.88-89)

The problem is that we have no way of knowing which act-individuals satisfy the existential quantifier and which do not. As Makinson puts it "It begins to look like a distinction without even the vaguest criterion of discrimination."(Makinson, 1981, p. 89). Moreover, saying that some act is such that is is permitted to be a case of smoking or forbidden to be a case of chewing gum is a peculiar way of speaking and certainly not how we speak about obligations and permissions in our day to day life. The fact that there is no natural way to read such formulas suggests that using act-individuals as the objects that the quantifiers range over is not very satisfying. I will instead opt for using individuals.

Our model is called a varying domain model because there are no restrictions on the domain function d and the domains of our accessible worlds are allowed to vary freely.

The set of worlds W, the accessibility relation R, the outer-domain D and the domain function d together make up the Kripke frame $\langle W, R, D, d\rangle$ of first-order modal models. By imposing frame conditions on models we can validate different kinds of formulas. As we have already seen, one of the frame conditions that is imposed on every kind of model that we consider in this thesis is the seriality condition. There are also some other important frame conditions that we will consider in this thesis.

We can impose the *increasing domains* frame condition by restricting the domain function such that every member of the actual world is also a member of every accessible world (sometimes called *monotonicity*):

Definition 11. *An increasing domain frame is a Kripke frame with the restriction that whenever Rww' holds, then also $D_w \subseteq D_{w'}$.*

Every model based on an increasing domain frame is an increasing domain model. The second option is to impose the *decreasing domains* frame condition by restricting the domain function such that every member of every accessible world is also a member of the actual world (sometimes called *anti-monotonicity*):

Definition 12. *A decreasing domain frame is a Kripke frame with the restriction that whenever Rww' holds, then also $D_{w'} \subseteq D_w$.*

Every model based on a decreasing domain frame is a decreasing domain model. The third option is the most restrictive one and demands that the domains of accessible worlds remain equal to the domain of the actual world. This will be called *constant domains*:

Definition 13. *A constant domain frame is a Kripke frame with the restriction that* *whenever Rww' holds, then also $D_w = D_{w'}$.*

Each of these frame conditions have important consequences for the kind of formulas that are valid (Fitting and Mendelsohn, 2012; Hughes and Cresswell, 1996). This will be discussed at length in section 3.4.

a is an assignment function that determines the extension of non-logical symbols (i.e. their reference) relative to each conceivable world. Members of $\mathcal{C} \cup \mathcal{V}$ have as their extension objects of D, while members of $\mathcal{P}^r$ have as their extension sets of n-tuples of $\wp(D^r)$. That there exists a set such that it is the proper extension of a predicate is guaranteed, in the case of a unary predicate, by using the power set of the domain and, in the case of predicates with rank $r > 1$, by using the power set of the r^{th}-Cartesian product.[4] The assignment function also determines the truth value of proposition symbols relative to a world by mapping them onto 0 (falsehood) or 1 (truth).

We can see that the assignment function a maps our variables and constants directly onto the set of all conceivable individuals. Our constants and variables will thus pick out the same conceivable individual in each conceivable world. They are what is called *rigid designators*. The predicates are relativised to a world but can contain individuals that do not exist at that world. This makes it possible to say that at w it is true that John is the great-grandfather of Marie although John has already passed away and so is no longer in the domain of w. The fact that our terms can refer to individuals that do not exist suggest that we are dealing with a free logic. In the next section I will show that this is indeed the case and what this entails.

3.2.2 Why we need free logics

As Garson (2001) notes, the adoption of varying domains practically forces us to adopt the principles of free logic.[5] Free logics are defined by Ermanno Bencivenga as follows:

> A free logic is a formal system of quantification theory, with or without identity, which allows for some singular terms in some circumstances to be thought of as denoting no existing object, and in which quantifiers are invariable thought of as having existential import. (Bencivenga, 2002, pp. 148-149)

As previously indicated, our varying domain models allow terms to refer to any individual in D and this assignment is not in any way bound to existence but only to mere conceivability. The varying domain models are also not at odds with Quine's dictum that "to be is to be the value of a bound variable" (Quine, 1939, p. 50) because the quantifiers range over D_w and so have existential import. These two characteristics of our varying domain models show us that we have in our

[4] That such a powerset can be constructed is guaranteed by the axiom of power set in ZFC that states that for every set x, there is a set $\wp(x)$ consisting precisely of all subsets of x.

[5] Free logics are originally introduced in Leonard (1956) and Lambert et al. (1967)

hands the semantics of a free logic. The fact that we end up with a free logic is no mistake because trying to preserve classical first-order logic forces one to take unsatisfying measures. To show why this is the case I will draw heavily upon Garson (2001).

First, Garson (2001) observes, $(\exists\alpha)(\alpha = \beta)$ is true at a world w in a model if the extension of β is in the domain D_w of that world. The problem is that $(\exists\alpha)(\alpha = \beta)$ is a theorem of classical first-order logic. If we accept this theorem then every term of the language must refer to an object that exists in every possible world. This, however, is at odds with the idea behind varying domain semantics because we want to allow that objects in one world do not exist in another. So one strategy to preserve classical rules is to simply eliminate terms, which is what Kripke's approach in Kripke (1963*a*) amounts to (Garson, 2001). The problem is that this does not give a satisfying account of terms as it simply eliminates them.

The second problem is that accepting the principles of first-order logic forces us to adopt increasing domains. The reason is that the converse Barcan formula ($\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$) is a theorem of first-order logic combined with the modal logic $\mathbf{K}$ (this derivation can be found at p. 557 of Braüner and Ghilardi (2007)). As we will see in section 3.4, every model must be based on an increasing domain frame to validate the converse Barcan formula. However, we do not want to impose this frame condition from the get-go because there aren't any established results concerning what kind of models are appropriate in deontic logics.

Suppose, however, that we do have good reason to adopt increasing domains then we will still run into trouble if we want to keep the classical principles of first-order logic. The classically valid formula $(\forall\alpha)A \supset A[\beta/\alpha]$ (known as *universal instantiation*) is, for example, not true in a world in a model if $\beta \notin D_w$ and A is P and $P = D_w$. One way in which to resolve the issue is to demand that the terms get assigned their extensions *locally*. That is, the extension of a term in w must be in D_w. However, as Garson (2001) contends, there are serious problems with this approach. First, it excludes terms such as "Pegasus" because their extension can not be any real object and so be in D_w. Second, because my terms are rigid they need to have the same extension in each world and so terms will need to have extensions that exist in every world, which undercuts the basic idea of varying domains. The second way in which we could resolve the issue is to demand that predicates get assigned their extensions locally.

Definition 14. *Local Predicates. Where a is the assignment function such that $a : \mathcal{P}^r \times W \to \wp(D^r)$ we ensure local predicates by demanding that for every arbitrary predicate $\pi^r \in \mathcal{P}^r$ and arbitrary world $w \in W$ we have $a(\pi^r, w) \in D^r_w$.*

This ensures that $\pi\alpha \supset (\exists\alpha)\pi\alpha$ is valid because the truth of $\pi\alpha$ ensures that $a(\alpha) \in D_w$. However, $\neg\pi\alpha \supset (\exists\alpha)\neg\pi\alpha$ is not valid. From the truth of $\neg\pi\alpha$ it does not follow that the extension of α is an existing object and so it does not follow that $(\exists\alpha)\neg\pi\alpha$ is true (Garson, 2001). Garson (2001) concludes: "Not only do we fail to validate the rule of existential generalization, but the valid principles cannot be expressed as axiom schemata. [...] In case we are using axioms and a rule of substitution of formulas for atoms, the problem re-emerges in the failure of the rule of substitution. Either way, the use of local predicates leads to serious formal difficulties"(Garson, 2001, pp. 276-277).

Given the anomalies that local predicates set us up with, it is surprising that Hilpinen and McNamara (2013) make use of local predicates when characterising the semantics of a quantified deontic logic. To see why this will not do as a starting point we can, for example, look at instances of the converse $\mathbf{P}$-Barcan formula ($(\exists\alpha)\mathbf{P}A \supset \mathbf{P}(\exists\alpha)A$) (this formula will be properly introduced in section 3.3). In section 3.3 I show that the converse $\mathbf{P}$-Barcan formula is syntactically equivalent to the converse Barcan formula ($\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$) and in section 3.4 I show that the converse Barcan formula is not valid on decreasing domain frames. However, when using local predicates, $(\exists x)\mathbf{P}Px \supset \mathbf{P}(\exists x)Px$ is valid on decreasing domain frames while $(\exists x)\mathbf{P}\neg Px \supset \mathbf{P}(\exists x)\neg Px$ is not valid.

Fact 1. *The formula $(\exists x)\mathbf{P}Px \supset \mathbf{P}(\exists x)Px$ is valid when using varying domains and local predicates.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \vDash (\exists x)\mathbf{P}Px$ and $\mathcal{M}, w \nvDash \mathbf{P}(\exists x)Px$. Because of $\mathcal{M}, w \vDash (\exists x)\mathbf{P}Px$ we know that among all models $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from D_w we will have at least one model $\mathcal{M}'$ such that $\mathcal{M}', w \vDash \mathbf{P}Px$. From this we know that in $\mathcal{M}'$ there is a w' such that Rww' at which we will have $\mathcal{M}', w' \vDash Px$. The only way in which $\mathcal{M}', w' \vDash Px$ can be true is if $a'(x) \in a'(P, w')$ and because of local predicates it is guaranteed that $a'(x) \in D_{w'}$ because every member of the extension of P must be in $D_{w'}$.

Because of $\mathcal{M}, w \nvDash \mathbf{P}(\exists x)Px$ we know by the negation clause that $\mathcal{M}, w \vDash \neg\mathbf{P}(\exists x)Px$. This formula is syntactically equivalent to $\mathbf{O}(\forall x)\neg Px$ given that $\mathbf{O}$ is interchangeable with $\neg\mathbf{P}\neg$ and $\forall$ with $\neg\exists\neg$ and double negation can be eliminated. This gives us $\mathcal{M}, w \vDash \mathbf{O}(\forall x)\neg Px$ from which we can infer that for all w' for which Rww' we will have in $\mathcal{M}$ that $\mathcal{M}, w' \vDash (\forall x)\neg Px$. This in turn establishes that in all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from $D_{w'}$ we will have $\mathcal{M}', w' \vDash \neg Px$ for all w'. However we already knew that there is a $\mathcal{M}'$ and a world w' in it for which we do have $\mathcal{M}', w' \vDash Px$ and for which we know that $a'(x) \in D_{w'}$. This leaves us with a contradiction. ■

Fact 2. *The formula $(\exists x)\mathbf{P}Px \supset \mathbf{P}(\exists x)Px$ is valid when imposing decreasing domain frames and local predicates.*

Proof. This fact follows *a fortiori* from the proof given for the varying domain case because decreasing domain models are a subset of varying domain models. ■

Fact 3. *The formula* $(\exists x)\mathbf{P}\neg Px \supset \mathbf{P}(\exists x)\neg Px$ *is not valid when imposing decreasing domain frames and local predicates.*

Proof. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_1, o_2\}$ and $d(w_2) = \{o_2\}$
5. $a(P, w_2) = \{o_2\}$

Now we can check whether the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$ and $\mathcal{M}, w_1 \nvDash \mathbf{P}(\exists x)\neg Px$. To check whether $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$ is true we have to take a look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ holds in one of these $\mathcal{M}'$. To determine whether $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ is true for one of these $\mathcal{M}'$ we have to search for a $\mathcal{M}'$ in which for at least one w' such that Rww' it is true that $\mathcal{M}', w' \vDash \neg Px$. Consider the $\mathcal{M}'$ and the assignment a' in it for which $a'(x) = o_1$. In this $\mathcal{M}'$ we know that there is only one w' for which Rww' namely w_2. We can see that $a(x) \notin a(P, w_2)$ and so we have $\mathcal{M}', w_2 \nvDash Px$ and by the negation clause $\mathcal{M}', w_2 \vDash \neg Px$. From this we know that $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ is true. This in turn establishes that there is a model $\mathcal{M}'$ such that $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ and so that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$ is true.

Now we have to show that $\mathcal{M}, w_1 \nvDash \mathbf{P}(\exists x)\neg Px$. I will do this by showing that its negation is true in $\mathcal{M}$, in other words that $\mathcal{M}, w_1 \vDash \neg\mathbf{P}(\exists x)\neg Px$ which is equivalent to showing that $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$ given that $\mathbf{O}$ is interchangeable with $\neg\mathbf{P}\neg$ and $\forall$ with $\neg\exists\neg$ and double negations can be eliminated. To show that $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$ is true we have to show that in $\mathcal{M}$ for all w' for which Rww' that $\mathcal{M}, w' \vDash (\forall x)Px$. We know that there is only one world w' for which Rww' namely w_2. To show that $\mathcal{M}, w_2 \vDash (\forall x)Px$ we have to look at all $\mathcal{M}'$, where a' differs at most from a concerning the value it assigns from D_{w_2} to x, and check if $\mathcal{M}', w_2 \vDash Px$ holds for all of these $\mathcal{M}'$. We know that for all $\mathcal{M}'$ it will be the case that $a'(x) = o_2$ because there is only one individual in D_{w_2} and so for all these $\mathcal{M}'$ we can see that it is true that $a'(x) \in a'(P, w_2)$. From this we infer that for all $\mathcal{M}'$ it is true that $\mathcal{M}', w_2 \vDash Px$. Because of this fact we know that $\mathcal{M}, w_2 \vDash (\forall x)Px$ is true. Now that we have checked all worlds w' for which Rww' we know that $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$. $\mathbf{O}(\forall x)Px$ is equivalent to $\neg\mathbf{P}(\exists x)\neg Px$ and so $\mathcal{M}, w_1 \vDash \neg\mathbf{P}(\exists x)\neg Px$. By the negation clause it follows that $\mathcal{M}, w_1 \nvDash \mathbf{P}(\exists x)\neg Px$. ■

These results show us that given decreasing domain frames and local predicates we can not even express all valid principles by using axiom schemata. That is, we cannot express the validity of $(\exists x)\mathbf{P}Px \supset \mathbf{P}(\exists x)Px$ by using $(\exists \alpha)\mathbf{P}A \supset \mathbf{P}(\exists \alpha)A$ as an axiom schema because $(\exists x)\mathbf{P}\neg Px \supset \mathbf{P}(\exists x)\neg Px$ is an instantiation of this axiom schema and, as I have shown, not valid. Using axioms and a rule of substitution will not do either as the problem will re-emerge as a failure of the rule of substitution. Because of these formal problems I conclude that working with local predicates is not a good basis to start from when trying to ascertain what kinds of models are appropriate within quantified deontic logic. Moreover, given the problems that occur when trying to preserve classical first-order logic I conclude that we should not try to preserve it and instead adopt the principles of free logic. Because we will be using free logics it will be useful to take a deeper look at what they encompass.

3.2.3 Interpreting quantified free logics

In section 3.2.1 I have defined the mathematical structure of the various kinds of models under consideration. The way in which the structure is defined determines exactly under what conditions the formulas of the language come out true. However, the job does not end there. We have to be able to intuitively grasp what it means that a certain formula is true and what it expresses in order to assess whether or not our models are actually useful if we apply them to natural language expressions and arguments. In particular, we have to get clear on the meaning of some basic formulas. Let us start with one of the most basic expressions.

Suppose we have $\mathcal{M}, w \vDash Pa$. It is true just in case the individual denoted by a is in the extension of the predicate P at the world w. The individual constant and predicate both get their values from the outer domain of the model. This means that the expression Pa can be true regardless of whether or not a exists at w. In fact, the extension of the predicate P, which is simply a set of objects taken from the powerset of the outer domain, needn't contain any individual that exists at w. These features make it clear that predicates and persons are very general concepts in our semantics. This is why I suggested earlier to think of the individuals "inhabiting" the outer domain as *conceivable individuals*. Suppose that a denotes Santa Claus and P is a predicate for "a bearded men". If Pa is true in w then that must mean that at w the imaginary individual we think of as "Santa Claus" should be thought of as having a beard. In talking about persons and features of those persons we have to abstract away from their actual existence. While at first sight this might feel kind of strange I do not think that it is necessarily a problematic feature of our semantics because it is designed to give us a better understanding of natural language and we seem to be talking about non-existent people and properties that nobody instantiates all the time.

Now suppose that P expresses an action. It might at first sight be puzzling how a conceivable individual can be in the extension of such a predicate at a world. However, just as it is not absurd to say that what it means to be Santa Claus is to be a bearded man independently of whether Santa Claus actually exists. It seems equally not absurd to assert that Santa Claus gives presents to children independently of whether he exists. The semantics thus commits us to the position that it is intelligible to give properties and ascribe actions to non-existent persons and objects.

$\mathcal{M}, w \vDash (\exists x)Px$ is true just in case an individual in the domain of w instantiates the property P. In other words: it tells us that a person who exists at w actually exhibits or instantiates the concept expressed by P. This explains why we cannot simply go from the truth of Pa to the truth of $(\exists x)Px$. The existential quantifier comes with real existential import; any existentially quantified formula will be false if we are dealing with a world with an empty-domain. On the other hand $\mathcal{M}, w \vDash (\forall x)Px$ does not come with the same kind of existential import. The reason is that a universally quantified formula is vacuously true when the domain is empty. However, it is still the case that a universally quantified formula can only genuinely tell us something about existing persons. If everything has a certain property at w then this means either that every existing individual at w has that property or that no individual exists at w.

$\mathcal{M}, w \vDash \mathbf{O}Pa$ is true if and only if in every accessible world w' the person denoted by a is in the extension of P at each of these w'. Recall that only deontically acceptable worlds are accessible. In other words: $\mathbf{O}Pa$ can only be true if Pa in each world w' does not violate what ought to be the case according to w. So, for example, suppose that it is obligatory at w that John babysits his sister Mary then this means that in each world that is acceptable from the point of view of w John babysits his sister. Conversely, a world in which John does not babysit Mary is not deontically acceptable and thus not accessible. Notice that it is entirely irrelevant whether or not John or Mary actually exist at those accessible worlds. Suppose it is true at w that John has to babysit his sister Mary and so we have $\mathcal{M}, w \vDash \mathbf{O}Pab$ where ‘a’ refers to John and ‘b’ to Mary. The truth of $\mathbf{O}Pab$ does not exclude there being acceptable worlds at which John and Mary or just one of them does not exist. The deontic operator does not discriminate between the existence and the non-existence of a person in the acceptable worlds. This is strange because the acceptable worlds are intended to be deontically relevant to the actual world and the people in them. Why should the acceptable world in which you are merely a non-existent person be of any normative relevance to what you ought to do in the actual world. This seems like an unintended consequence of the fact that we are forced to work with free logics.

Suppose that Mary does not exist in one of those acceptable worlds w', what then does it mean that John is at w' in the extension of P? I don’t think there is a straightforward answer to this question but nevertheless an answer we need if we want to make sense of this kind of semantics. An attempt to give an answer to this question might look something like this. When we are reasoning about what a particular person ought to do we can simply imagine her doing a particular action or having a particular property and then ask ourselves whether that would be allowed or would be necessary given her obligations without explicitly taking into account her existence. What our semantics would commit us to is, in a sense, that we can speak intelligibly about what a person ought to do or be while at the same time “bracketing” his or hers existence. This is the sort of interpretation that Martin Mose Bentzen seems to have in mind in his thesis "*Deontic Logics and Imperative Logics - A Historical Overview of Normative Logics from Ernst Mally to Defeasible Deontic Logic and a New System*". He presents a counter model and gives an interpretation of that model:

> Informally if Andrea(a)and Bert(b) are persons then Andrea exists in the actual world Reality (Γ) and Bert exists in the perfect world Utopia(Ω). Since Utopia is perfect nobody there is required to go to jail, because nobody there commits crimes. Now in the horrible actual world that Reality is, Andrea does not go to jail (we do not have $\Gamma \vDash Q(a)$), because here money rules, legal systems are corrupt, she is a white, rich protestant and she has a devious lawyer. However, in the perfect world Andrea would be required to go to jail, since she has committed a crime. (When the saint Bert discusses Andrea's case with himself, he comes to the conclusion that if people like Andrea existed, they would have to go to jail. Then he goes to bed, happily assured that they don't). (Bentzen, 2004, p. 55)

I will not reproduce the specifics of this model because it is not relevant here. What is relevant is the fact that he uses a model and a perfect (acceptable) world in which Andrea does not exist and is in the extension of Q where Q is a predicate for going to jail. He interprets this as as saying "if people like Andrea existed, they would have to go to jail.". This kind of interpretation is not very satisfying, however, because it seems to avoid the problem by not taking the semantics at face value. If we were to hypothetically think about how someone ought to behave we would most likely imagine how that person would behave in the ideal situation and, surely, it does not make much sense to also take into account ideal situations in which that person does not exist.

A way in which to go forward is to modify the clause of the deontic operator to give us a more satisfying deontic operator. We can do this by modifying the standard deontic operator clause as follows:

Definition 15. *Van Benthem Clause: $\mathcal{M}, w \vDash \mathbf{O}A$ iff $\mathcal{M}, w' \vDash A$ for all w' for which Rww' and $a(\alpha) \in D_{w'}$ for every α free in A.*

This clause does discriminate between existing and non-existing people because to determine the truth of a formula such as $\mathbf{O}Pa$ it will not take into account acceptable worlds at which the denotation of a does not exist. I first encountered this kind of clause in Hilpinen (2002) who refers to Van Benthem (1990). For future reference, I will name it after Johan Van Benthem because his discussion of it in "*A manual of intensional logic*" is the oldest reference I could find, although I do not know whether he is the first to describe it. As I will demonstrate in section 3.4, the adoption of this clause makes a major difference for the conditions under which some of the important formulas are valid. This gives us additional reason to keep it in mind if it proves to be a better fit with the formulas that we desire to be valid.

$\mathcal{M}, w \vDash (\exists x)\mathbf{O}Px$ expresses that there actually is someone existing at w for whom it is true that at each acceptable world s/he is in the extension of P. Again, we cannot go from the truth of $\mathbf{O}Pa$ to the truth of $(\exists x)\mathbf{O}Px$. We can only do so if we know that someone actually existing at w has the obligation to P. The move from $\mathbf{O}Pa$ to $(\exists x)\mathbf{O}Px$ thus does not come with additional normative content. $(\forall x)\mathbf{O}Px$, if it is not vacuously true, says something about a set of people, i.e. that they all have an obligation to P. Contrary to $\mathcal{M}, w \vDash (\exists x)\mathbf{O}Px$, $\mathcal{M}, w \vDash \mathbf{O}(\exists x)Px$ is more of an explicit normative statement. It asserts that at each acceptable world there has to exist someone who is in the extension of P. An important difference between $(\exists x)\mathbf{O}Px$ and $\mathbf{O}(\exists x)Px$ is that for the first one to be true there has to be at least some unique person such that s/he is in the extension of P at each acceptable world while $\mathbf{O}(\exists x)Px$ asserts that at each accessible world there has to be a person that is in the extension of P but it doesn't need to be the same person in each accessible world.

A formula such as $\mathbf{O}(\forall x)Px$ demands that every person in the domain of every acceptable world P's. This suggests that we have to do with a general norm that does not admit of any exceptions. $\mathbf{P}(\exists x)Px$ seems to be saying something to the effect of: a state of affairs in which someone P's is an acceptable situation and it doesn't matter who that someone is. $\mathbf{P}(\forall x)Px$ is an even stronger formula that says that a situation in which everyone P's is acceptable.

This concludes my brief discussion of the semantic meaning of some important basic formulas of the language. These formulas will be discussed in more depth in chapter 4 where we will give them a more explicit interpretation. First we will discuss what is meant by "the syntax of a logic" and give some options of how the syntax of a quantified deontic logic might look like.

3.3 The syntax

The syntax of a logic determines the way in which formulas are related to each other. One way to determine the relation between formulas is with an axiomatic system. In an axiomatic system we stipulate which formulas are the axioms and the rules by which to derive other formulas from them. The formulas we are able to derive from our axioms are the theorems of the logic. I will write $\vdash A$ to indicate that A is an axiom or a theorem. To characterise various axiomatic systems I will use axiom schemata. Each instantiation of an axiom schema is an axiom. To characterize quantified deontic logics I will use Standard Deontic Logic (as defined in Hilpinen and McNamara (2013)) as a starting point and then expand on it.

3.3.1 Standard Deontic Logic

First there are the axiom schemes and rules of Standard Deontic Logic:

TAUT All propositional tautologies of the language

KD $\mathbf{O}(A \supset B) \supset (\mathbf{O}A \supset \mathbf{O}B)$

DD $\mathbf{O}A \supset \neg\mathbf{O}\neg A$

MP If $\vdash A$ and $\vdash A \supset B$ then $\vdash B$

RND If $\vdash A$ then $\vdash \mathbf{O}A$

Every propositional tautology is an axiom, **KD** is the deontic counterpart to the **K** axiom (it is named K after Kripke). **DD** entails that every obligation is also permissible (given the definition of the **P**-operator) and enforces deontic consistency. The two rules are *modus ponens* (**MP**) and *deontic necessitation* (**RND**). A modal logic is considered *normal* if its set of theorems contains all propositional tautologies, every instance of the axiom schema **K** and is closed under *modus ponens* and *necessitation*. We can see that **SDL** is an example of a normal modal logic.

Definition 16. **SDL** *is the logic obtained by adding to the propositional tautologies of the language the axiom schemata* **KD** *and* **DD***, and the rules* **MP** *and* **RND**.

3.3.2 Quantified free deontic Logic

To characterize the predicative fragment we need some additional schemes and rules (adopted from Corsi (2002*b*).

Reflexivity $\beta = \beta$

Substitutivity $(\alpha = \beta \wedge A[\alpha/\gamma]) \supset A[\beta/\gamma]$

Necessary distinctness $(\alpha \neq \beta) \supset \Box(\alpha \neq \beta)$

$\forall$ **Distributivity** $(\forall\alpha)(A \supset B) \supset ((\forall\alpha)A \supset ((\forall\alpha)A \supset (\forall\alpha)B))$

$\forall$ **Permutation** $(\forall\alpha)(\forall\beta)A \supset (\forall\beta)(\forall\alpha)A$

Free $\forall$ **elimination** $(\forall\beta)(\forall\alpha)(A \supset A[\beta/\alpha])$

Vacuous Quantification $A \supset (\forall\alpha)A$ where α not free in A.

Universal Generalization If $\vdash A$ then $\vdash (\forall\alpha)A$.

Reflexivity and **Substitutivity** form a standard axiomatisation of equality in first-order logic. **Necessary distinctness** is needed to reflect the fact that our variables and constants are *rigid designators*. However, we do not have traditional quantifier elimination ($(\forall\alpha)A \supset A[\beta/\alpha]$) and introduction (if $(A \supset A[\beta/\alpha]$ then $A \supset (\forall\alpha)A$). The reason is that the extension of a variable can denote a non-existing object in which case it will not be a member of the quantifier domain. This allows, for example, that the antecedent of $(\forall\alpha)A \supset A[\beta/\alpha]$ is true while its consequent is false. To prevent this from happening we have to add a "guard". By quantifying over the variable β we make sure that it designates only objects that exist which gives us **Free $\forall$ elimination.** This gives us an axiomatic system for the quantifiers that is similar to the usual axiomatic characterisation of free logics (this type of axiomatisation is first introduced in Kripke (1963*a*) and discussed in (Braüner and Ghilardi, 2007; Hughes and Cresswell, 1996; Fine, 1983; Fitting and Mendelsohn, 2012)).

Definition 17. **QFD** *(quantified free deontic logic) is the logic obtained by adding to* ***SDL*** *the axiom schemata* **Reflexivity***,* **Substitutivity***,* **Necessary distinctness***,* **$\forall$ Distributivity***,* **$\forall$ Permutation***,* **Free $\forall$ elimination** *and* **Vacuous Quantification***, and the rule* **Universal Generalization**.

3.3.3 The syntactic relation between the quantifiers and the deontic operators

The following four axiom schemes are able to determine the syntactic relation between the quantifiers and the deontic operators:

BF $(\forall\alpha)\mathbf{O}A \supset \mathbf{O}(\forall\alpha)A$

CBF $\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$

GF $(\exists\alpha)\mathbf{O}A \supset \mathbf{O}(\exists\alpha)A$

CGF $\mathbf{O}(\exists\alpha)A \supset (\exists\alpha)\mathbf{O}A$

The first two are the Barcan formula (**BF**) and the converse Barcan formula (**CBF**) originally introduced by Ruth Barcan Marcus in Barcan (1946). The second two are the Ghilardi formula (**GF**) and the converse Ghilardi formula (**CGF**).[6] These formulas play a crucial role within quantified modal logics and will thus be an important object of study in this thesis. We can add each of these formulas or a combination of them to our base logic **QFD** and each of these options will give us a different logic. The way in which these options relate to the different kinds of models will become clear in the section ahead. I will define each of these possible logics but I will exclude logics that contain **CGF** because as we will see in section 3.4, this formula is not valid on any of the models we will take into account. This, I will argue in section 4.3, is not a mistake.[7] I will also exclude the logic $\mathbf{QFD}+\mathbf{CBF}+\mathbf{GF}$ and $\mathbf{QFD}+\mathbf{BF}+\mathbf{CBF}+\mathbf{GF}$ because **GF** is a theorem of $\mathbf{QFD}+\mathbf{CBF}$ and so it would be redundant to add it as an axiom schema if we already have **CBF**.

Fact 4. *The Ghilardi formula is a theorem of* $\mathbf{QFD}+\mathbf{CBF}$.

6 This is the name given to it in Corsi (2002*b*), Gochet and Gribomont (2006) and Calardo (2013). It is named after Silvio Ghilardi, a contemporary italian mathematician and logician.
7 See Van Benthem (2010) page 307 for the semantic conditions needed to validate this formula.

Proof. Some minor steps have been omitted:

1. $(\forall x)(Px \supset (\exists x)Px)$ (Theorem instance)
2. $\mathbf{O}(\forall x)(Px \supset (\exists x)Px)$ (1; $\mathbf{RND}$)
3. $(\forall x)\mathbf{O}(Px \supset (\exists x)Px)$ (2; $\mathbf{CBF}$)
4. $(\forall x)(\mathbf{O}Px \supset \mathbf{O}(\exists x)Px)$ (3; $\mathbf{KD}$)
5. $(\forall x)(\mathbf{O}Px \supset \mathbf{O}(\exists x)Px) \supset ((\exists x)\mathbf{O}Px \supset \mathbf{O}(\exists x)Px))$ (Theorem instance)
6. $(\exists x)\mathbf{O}Px \supset \mathbf{O}(\exists x)Px)$ (4, 5; $\mathbf{MP}$)

The proof makes use of two theorems of $\mathbf{QFD}$ namely $(\forall\alpha)(A[\alpha] \supset (\exists\beta)A[\beta/\alpha])$ (line 1) and $(\forall\alpha)(A[\alpha] \supset B) \supset ((\exists\alpha)A[\alpha] \supset B))$ where α is not free in B (line 5).[8] This proof can be found in Fitting (1999). The fact that $\mathbf{GF}$ is a theorem of $\mathbf{QFD}+\mathbf{CBF}$ should be kept in mind because it will be important later on when we discuss the effect of the Van Benthem clause on the semantics.

Definition 18. $\mathbf{QFD}+\mathbf{BF}$ *is the logic obtained by adding to* $\mathbf{QFD}$ *the axiom schema* $\mathbf{BF}$.

Definition 19. $\mathbf{QFD}+\mathbf{CBF}$ *is the logic obtained by adding to* $\mathbf{QFD}$ *the axiom schema* $\mathbf{CBF}$.

Definition 20. $\mathbf{QFD}+\mathbf{GF}$ *is the logic obtained by adding to* $\mathbf{QFD}$ *the axiom schema* $\mathbf{GF}$.

Definition 21. $\mathbf{QFD}+\mathbf{BF}+\mathbf{CBF}$ *is the logic obtained by adding to* $\mathbf{QFD}$ *the axiom schemata* $\mathbf{BF}$ *and* $\mathbf{CBF}$.

Definition 22. $\mathbf{QFD}+\mathbf{BF}+\mathbf{GF}$ *is the logic obtained by adding to* $\mathbf{QFD}$ *the axiom schemata* $\mathbf{BF}$ *and* $\mathbf{GF}$.

Let us now take a look at all the possibilities with respect to formulas containing deontic operators and quantifiers. We can observe that there are 8 possible permutations in total: $(\forall\alpha)\mathbf{O}A$, $\mathbf{O}(\forall\alpha)A$, $\mathbf{P}(\exists\alpha)A$, $(\exists\alpha)\mathbf{P}A$, $(\exists\alpha)\mathbf{O}A$, $\mathbf{O}(\exists\alpha)A$, $\mathbf{P}(\forall\alpha)A$, $(\forall\alpha)\mathbf{P}A$. These possible permutations stand in the following relation to each other:

[8] Corsi (2002*b*) mentions these two as theorems of her logic $\mathbf{Q}^{\circ}_{=}.\mathbf{K}$ which is axiomatically defined the same as $\mathbf{QFD}$ minus the $\mathbf{DD}$ axiom scheme.

Barcan formula	$(\forall\alpha)\mathbf{O}A \supset \mathbf{O}(\forall\alpha)A$	$\mathbf{P}$-Barcan formula[9]	$\mathbf{P}(\exists\alpha)A \supset (\exists\alpha)\mathbf{P}A$
converse Barcan formula	$\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$	converse $\mathbf{P}$-Barcan formula	$(\exists\alpha)\mathbf{P}A \supset \mathbf{P}(\exists\alpha)A$
Ghilardi formula	$(\exists\alpha)\mathbf{O}A \supset \mathbf{O}(\exists\alpha)A$	Buridan formula[10]	$\mathbf{P}(\forall\alpha)A \supset (\forall\alpha)\mathbf{P}A$
converse Ghilardi formula	$\mathbf{O}(\exists\alpha)A \supset (\exists\alpha)\mathbf{O}A$	converse Buridan formula	$(\forall\alpha)\mathbf{P}A \supset \mathbf{P}(\forall\alpha)A$

Because the interrelations between these formulas are important for what is to come I will demonstrate for each row the equivalence between the formula in the second column and the one in the fourth column. The steps I give are given in just one direction, but they can equivalently be given in reverse order for the other direction.

Fact 5. *The Barcan formula* $(\forall\alpha)\mathbf{O}A \supset \mathbf{O}(\forall\alpha)A$ *is syntactically equivalent to the* $\mathbf{P}$*-Barcan formula* $\mathbf{P}(\exists\alpha)A \supset (\exists\alpha)\mathbf{P}A$.

Proof. We start with $(\forall\alpha)\mathbf{O}A \supset \mathbf{O}(\forall\alpha)A$. Replace every $(\forall\alpha)$ by $\neg(\exists\alpha)\neg$ and every $\mathbf{O}$ by $\neg\mathbf{P}\neg$. This leaves us with $\neg(\exists\alpha)\neg\neg\mathbf{P}\neg A \supset \neg\mathbf{P}\neg\neg(\exists\alpha)\neg A$. Now we eliminate double negations, which gives us: $\neg(\exists\alpha)\mathbf{P}\neg A \supset \neg\mathbf{P}(\exists\alpha)\neg A$. Lastly, we use contraposition[11] and eliminate double negations to get: $\mathbf{P}(\exists\alpha)\neg A \supset (\exists\alpha)\mathbf{P}\neg A$. ■

Fact 6. *The converse Barcan formula* $\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$ *is syntactically equivalent to the converse* $\mathbf{P}$*-Barcan formula* $(\exists\alpha)\mathbf{P}A \supset \mathbf{P}(\exists\alpha)A$.

Proof. We start with $\mathbf{O}(\forall\alpha)A \supset (\forall\alpha)\mathbf{O}A$. Replace every $(\forall\alpha)$ by $\neg(\exists\alpha)\neg$ and every $\mathbf{O}$ by $\neg\mathbf{P}\neg$. This leaves us with $\neg\mathbf{P}\neg\neg(\exists\alpha)\neg A \supset \neg(\exists\alpha)\neg\neg\mathbf{P}\neg A$. Now we eliminate double negations, which gives us: $\neg\mathbf{P}(\exists\alpha)\neg A \supset \neg(\exists\alpha)\mathbf{P}\neg A$. Lastly, we use contraposition and eliminate double negations to get: $(\exists\alpha)\mathbf{P}\neg A \supset \mathbf{P}(\exists\alpha)\neg A$. ■

Fact 7. *The Ghilardi formula* $(\exists\alpha)\mathbf{O}A \supset \mathbf{O}(\exists\alpha)A$ *is syntactically equivalent to the Buridan formula* $\mathbf{P}(\forall\alpha)A \supset (\forall\alpha)\mathbf{P}A$.

[9] This formula is usually also called a Barcan formula. To distinguish it from its counterpart in terms of obligations I will use the name $\mathbf{P}$-Barcan formula.
[10] It was given its name by Alvin Plantinga (Plantinga, 1974) after the medieval logician Jean Buridan.
[11] Contraposition is the rule that allows you to infer $\neg B \supset \neg A$ from $A \supset B$.

Proof. We start with $(\exists\alpha)\mathbf{O}A \supset \mathbf{O}(\exists\alpha)A$. Replace every $(\forall\alpha)$ by $\neg(\exists\alpha)\neg$ and every $\mathbf{O}$ by $\neg\mathbf{P}\neg$. This leaves us with $\neg(\forall\alpha)\neg\neg\mathbf{P}\neg A \supset \neg\mathbf{P}\neg\neg(\forall\alpha)\neg A$. Now we eliminate double negations, which gives us: $\neg(\forall\alpha)\mathbf{P}\neg A \supset \neg\mathbf{P}(\forall\alpha)\neg A$. Lastly, we use contraposition and eliminate double negations to get: $\mathbf{P}(\forall\alpha)\neg A \supset (\forall\alpha)\mathbf{P}\neg A$. ■

Fact 8. *The converse Ghilardi formula* $\mathbf{O}(\exists\alpha)A \supset (\exists\alpha)\mathbf{O}A$ *is syntactically equivalent to the converse Buridan formula* $(\forall\alpha)\mathbf{P}A \supset \mathbf{P}(\forall\alpha)A$.

Proof. We start with $\mathbf{O}(\exists\alpha)A \supset (\exists\alpha)\mathbf{O}A$. Replace every $(\forall\alpha)$ by $\neg(\exists\alpha)\neg$ and every $\mathbf{O}$ by $\neg\mathbf{P}\neg$. This leaves us with $\neg\mathbf{P}\neg\neg(\forall\alpha)\neg A \supset \neg(\forall\alpha)\neg\neg\mathbf{P}\neg A$. Now we eliminate double negations, which gives us: $\neg\mathbf{P}(\forall\alpha)\neg A \supset \neg(\forall\alpha)\mathbf{P}\neg A$. Lastly, we use contraposition and eliminate double negations to get: $(\forall\alpha)\mathbf{P}\neg A \supset \mathbf{P}(\forall\alpha)\neg A$. ■

3.4 The relation between syntax and semantics

One of the aims of the logician is to make sure there is an intimate connection between syntax and semantics. As Kant's dictum goes "syntax without semantics is empty, but semantics without syntax is blind."(Woleński, 2012, p. 589) on which Jan woleński expands "...the Kantian metaphor that syntax without semantics is empty, but semantics without syntax is blind, means that the preciseness of calculus sharpens the semantic eye, although semantics brings the content into correct formulas."(Woleński, 2012, p. 595).

One way in which to connect the two is by relating the provable formulas within the syntax of a logic (i.e. its axioms and theorems) to the valid formulas of its semantics. A natural question to ask is whether every formula that is provable is also valid and every valid formula also provable. If every provable formula is valid the logic is considered *sound*. If every valid formula has a proof, it is considered *complete*.

One example of the importance of this connection is that if our logic is sound and complete we can show that it is impossible to derive a contradiction from our axioms by showing that there exists a model of them. This is due to the fact that an inconsistent set of formulas does not have a model. This is not possible with purely syntactic means because, given a particular set of premises, it can not be shown that it is impossible to derive an inconsistency from it.

If we want our syntax and semantics in alignment, we have to carefully consider which axioms and rules we want in our logic and how to make sure our models make them and only them valid. To further illuminate this link we will take a look at the 8 formulas introduced in the previous section and see what kind of semantics we need to validate or invalidate them.

I will first show that if we use varying domain models and do not restrict the domain function in any way we will have models that make the Barcan formula, the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula false. Which is the same as saying that those formulas are not valid when using varying domain models. We have already established that the $\mathbf{P}$-Barcan formula, the converse $\mathbf{P}$-Barcan formula, the Buridan formula and the converse Buridan formula are their syntactic equivalents and so they are equally not valid on varying domain models. I will also show that if we use the Van Benthem clause instead of the standard deontic operator clause the converse Barcan formula is valid when using varying domain models.

3.4.1 Varying domain models

Fact 9. *The Barcan formula is not valid when using varying domain models and the standard deontic operator clause or the Van Benthem clause.*

Proof. To show that the Barcan formula is not valid when using varying domain models we need to find one possible varying domain model that makes its antecedent true and consequent false. This counter model works for both the standard deontic operator clause and the Van Benthem clause.

Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_1\}$ and $d(w_2) = \{o_1, o_2\}$
5. $a(P, w_1) = a(P, w_2) = \{o_1\}$

Now we can check whether the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash (\forall x)\mathbf{O}Px$ and $\mathcal{M}, w_1 \nvDash \mathbf{O}(\forall x)Px$. To determine whether $\mathcal{M}, w \vDash (\forall x)\mathbf{O}Px$ we have to take a look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ holds in all these $\mathcal{M}'$ (clause 7 of the valuation). For every $\mathcal{M}'$ and the assignment a' in it, it will be the case that $a'(x) = o_1$ because o_1 is the only individual in D_{w_1} and D_{w_1} remains the same for all $\mathcal{M}'$. To determine whether $\mathcal{M}', w_1 \vDash \mathbf{O}Px$, we look at all w' for which Rw_1w' and check whether $\mathcal{M}', w' \vDash Px$ for all w'. Because every model model $\mathcal{M}'$ will be identical to $\mathcal{M}$ we know that in every $\mathcal{M}'$ we have w_1 and only one world for which Rw_1w' namely w_2 and that for w_2 we have $a'(x) \in a'(P, w_2)$ and so we know that $\mathcal{M}', w' \vDash Px$ for all w'. This allows us to conclude that $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ holds for all such $\mathcal{M}'$ and so $\mathcal{M}, w_1 \vDash (\forall x)\mathbf{O}Px$.

Now we have to show that $\mathcal{M}, w_1 \nvDash \mathbf{O}(\forall x)Px$. I will do this by showing that its negation is true in M, in other words that $\mathcal{M}, w_1 \vDash \neg\mathbf{O}(\forall x)Px$ which is equivalent to showing that $\mathcal{M}, w_1 \vDash \mathbf{P}(\exists x)\neg Px$ given that $\mathbf{O}$ is interchangeable with $\neg\mathbf{P}\neg$ and $\forall$ with $\neg\exists\neg$ and double negation can be eliminated. To show that $\mathcal{M}, w_1 \vDash \mathbf{P}(\exists x)\neg Px$ is true we have to show that in $\mathcal{M}$ there is a w' for which Rww' and $\mathcal{M}, w' \vDash (\exists x)\neg Px$. We know that there is only one world w' for which Rw_1w' namely w_2. To show that $\mathcal{M}, w_2 \vDash (\exists x)\neg Px$ we have to look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_2} to x, and check if $\mathcal{M}', w_2 \vDash \neg Px$ holds in one of these $\mathcal{M}'$. We know that there must be a $\mathcal{M}'$ for which $a'(x) = o_2$ and so for this $\mathcal{M}'$ it is true that $a'(x) \notin a'(P, w_2)$. From this we infer that there is a $\mathcal{M}'$ such that $\mathcal{M}', w_2 \nvDash Px$ and via the negation clause that for this $\mathcal{M}'$, $\mathcal{M}', w_2 \vDash \neg Px$ holds. Because of this fact we know that $\mathcal{M}, w_2 \vDash (\exists x)\neg Px$ is true. We also already knew that there is a world w for which Rww_2 in $\mathcal{M}$, namely w_1 and so we now know that $\mathcal{M}, w_1 \vDash \mathbf{P}(\exists x)\neg Px$. $\mathbf{P}(\exists x)\neg Px$ is equivalent to $\neg\mathbf{O}(\forall x)Px$ and so $\mathcal{M}, w_1 \vDash \neg\mathbf{O}(\forall x)Px$. By the negation clause it follows that $\mathcal{M}, w_1 \nvDash \mathbf{O}(\forall x)Px$. ■

Fact 10. *The converse Barcan formula is not valid when using varying domain models and the standard deontic operator clause.*

Proof. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_1, o_2\}$ and $d(w_2) = \{o_1\}$
5. $a(P, w_1) = a(P, w_2) = \{o_1\}$

Now we can check whether the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$ and $\mathcal{M}, w_1 \nvDash (\forall x)\mathbf{O}Px$. To check whether $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$ we have to consider all worlds w' for which Rww' and see whether $\mathcal{M}, w' \vDash (\forall x)Px$ for all such w'. We can see that there is only one world we have to check, namely w_2. To ascertain whether $\mathcal{M}, w_2 \vDash (\forall x)Px$ holds we have to look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_2} to x, and check if $\mathcal{M}', w_2 \vDash Px$ holds in all these $\mathcal{M}'$. There is only one individual, o_1, in D_{w_2} and so this will be the individual that gets assigned to x in every $\mathcal{M}'$. We can see that in $\mathcal{M}$ it holds that $a(x) \in a(P, w_2)$ and we know that the assignment function a does not change with respect to the extension of P in all the models $\mathcal{M}'$. From this it follows that for all $\mathcal{M}'$ it is true that $\mathcal{M}', w_2 \vDash Px$. Because of this fact we know that $\mathcal{M}, w_2 \vDash (\forall x)Px$. We have checked every w' for which Rww' because there only was one to check and so now we can conclude that $\mathcal{M}, w_1 \vDash \mathbf{O}(\forall x)Px$.

Now we have to prove that $\mathcal{M}, w_1 \nvDash (\forall x)\mathbf{O}Px$. I will do this by showing that its negation is true in $\mathcal{M}$, namely that $\mathcal{M}, w_1 \vDash \neg(\forall x)\mathbf{O}Px$ which is equivalent to $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$. To check whether $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$ we have to take a look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ holds in one of these $\mathcal{M}'$. To check whether $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ holds for one of these models we have to search among these models a model $\mathcal{M}'$ and a world w' for which Rww' and $\mathcal{M}', w' \vDash \neg Px$. In all these models $\mathcal{M}'$ there is but one world w' to check for which Rww', namely w_2. We know that there must be a model $\mathcal{M}'$ in which $a(x) = o_2$ and in that case $a(x) \notin a(P, w_2)$ and so we will have a model $\mathcal{M}'$ for which $\mathcal{M}', w_2 \nvDash Px$ and by the negation clause we have $\mathcal{M}', w_2 \vDash \neg Px$. This result establishes that in this $\mathcal{M}'$ we have $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ and subsequently that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{P}\neg Px$. Because of the equivalence between $(\exists x)\mathbf{P}\neg Px$ and $\neg(\forall x)\mathbf{O}Px$ we know that $\mathcal{M}, w_1 \vDash \neg(\forall x)\mathbf{O}Px$ and by the negation clause that $\mathcal{M}, w_1 \nvDash (\forall x)\mathbf{O}Px$. ■

Fact 11. *The converse Barcan formula is valid when using varying domain models and the Van Benthem Clause.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \vDash \mathbf{O}(\forall x)Px$ and $\mathcal{M}, w \nvDash (\forall x)\mathbf{O}Px$. Because of $\mathcal{M}, w \vDash \mathbf{O}(\forall x)Px$ we know that for every w' for which Rww' that $\mathcal{M}, w' \vDash (\forall x)Px$. From this we can infer that for all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from $D_{w'}$ and for all worlds w' it holds that $\mathcal{M}', w' \vDash Px$.

Because of $\mathcal{M}, w \nvDash (\forall x)\mathbf{O}Px$ we know by the negation clause that $\mathcal{M}, w \vDash \neg(\forall x)\mathbf{O}Px$ which is equivalent to $\mathcal{M}, w \vDash (\exists x)\mathbf{P}\neg Px$. Because of this we know that there is a model $\mathcal{M}'$ for which the assignment function a' differs at most from a with respect to the value it assigns to x from D_w for which $\mathcal{M}', w \vDash \mathbf{P}\neg Px$. From this we can infer that in this model $\mathcal{M}'$ there is a world w' for which Rww' and $\mathcal{M}', w' \vDash \neg Px$.

By the first part of the proof we know that for all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from $D_{w'}$ and for all worlds w' it holds that $\mathcal{M}', w' \vDash Px$. Hence, for every a' in every such $\mathcal{M}'$ it holds that $a'(x) \in a'(P, w')$ where $a'(x) \in D_{w'}$. By the second part of the proof we know that there is a model $\mathcal{M}'$ and a world w' in it for which $\mathcal{M}', w' \vDash \neg Px$ which tells us that in this $\mathcal{M}'$ it is the case that $a'(x) \notin a'(P, w')$ where $a'(x) \in D_w$. Because of the Van Benthem clause we know that this $a'(x)$ must also be in $D_{w'}$ and so $a'(x) \in D_{w'}$ but for every $a'(x) \in D_{w'}$ it holds that $a'(x) \in a'(P, w')$ which leaves us with a contradiction. ■

Fact 12. *The Ghilardi formula is not valid when using varying domain models and the standard deontic operator clause or the Van Benthem clause.*

Proof. In order to proof this fact I will produce a counter model that works for both the standard deontic clause and the Van Benthem clause. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2, w_3\}$

2. $R = \{\langle w_1, w_2\rangle, \langle w_2, w_2\rangle, \langle w_1, w_3\rangle, \langle w_3, w_3\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_1, o_2\}$ and $d(w_2) = \{o_2\}$ and $d(w_3) = \{o_1\}$
5. $a(P, w_2) = a(P, w_3) = \{o_1\}$

Now we can check whether the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{O}Px$ and $\mathcal{M}, w_1 \nvDash \mathbf{O}(\exists x)Px$. To check whether $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{O}Px$ is true we have to take a look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ holds in one of these $\mathcal{M}'$. To determine whether $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ is true for one of these $\mathcal{M}'$ we have to search for a $\mathcal{M}'$ in which for every w' such that Rw_1w' it is true that $\mathcal{M}', w' \vDash Px$. Consider the $\mathcal{M}'$ and the assignment a' in it for which $a'(x) = o_1$.

If we use the Van Benthem Clause there is in this $\mathcal{M}'$ only one w' for which Rw_1w' and $a'(x) \in D_{w'}$ namely w_3. We can see that $a(x) \in a(P, w_3)$ and so we have $\mathcal{M}', w_3 \vDash Px$. Because w_3 was the only accessible world to check we now know that $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ is true. This in turn establishes that there is a model $\mathcal{M}'$ such that $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ and so that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{O}Px$ is true.

If we use the Standard Deontic operator Clause there is in this $\mathcal{M}'$ two w' for which Rw_1w' namely w_2 and w_3. We can see that $a(x) \in a(P, w_2)$ and so we have $\mathcal{M}', w_2 \vDash Px$. We can also see that $a(x) \in a(P, w_3)$ and so we have $\mathcal{M}', w_3 \vDash Px$. Because w_2 and w_3 were the only accessible worlds to check we know that $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ is true. This in turn establishes that there is a model $\mathcal{M}'$ such that $\mathcal{M}', w_1 \vDash \mathbf{O}Px$ and so that $\mathcal{M}, w_1 \vDash (\exists x)\mathbf{O}Px$ is true.

Now we have to show that $\mathcal{M}, w_1 \nvDash \mathbf{O}(\exists x)Px$. I will do this by showing that its negation is true in $\mathcal{M}$, in other words that $\mathcal{M}, w_1 \vDash \neg\mathbf{O}(\exists x)Px$ which is equivalent to showing that $\mathcal{M}, w_1 \vDash \mathbf{P}(\forall x)\neg Px$ given that $\mathbf{O}$ is interchangeable with $\neg\mathbf{P}\neg$ and $\forall$ with $\neg\exists\neg$ and double negation can be eliminated. To show that $\mathcal{M}, w_1 \vDash \mathbf{P}(\forall x)\neg Px$ is true we have to show that in $\mathcal{M}$ there is a w' for which Rw_1w' and $\mathcal{M}, w' \vDash (\forall x)\neg Px$. Let us consider w_2. To show that $\mathcal{M}, w_2 \vDash (\forall x)\neg Px$ we have to look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_2} to x, and check if $\mathcal{M}', w_2 \vDash \neg Px$ holds for all of these $\mathcal{M}'$. We know that for all $\mathcal{M}'$ it will be the case that $a'(x) = o_2$ because there is only one individual in D_{w_2} and so for all these $\mathcal{M}'$ we can see that it is true that $a'(x) \notin a'(P, w_2)$. From this we infer that for all $\mathcal{M}'$ it is true that $\mathcal{M}', w_2 \nvDash Px$ and via the negation clause that for all these $\mathcal{M}'$ it holds that $\mathcal{M}', w_2 \vDash \neg Px$. Because of this fact we know that $\mathcal{M}, w_2 \vDash (\forall x)\neg Px$ is true. Now that we have found a world w' for which Rw_1w' and $\mathcal{M}, w' \vDash (\forall x)\neg Px$ we know that $\mathcal{M}, w_1 \vDash \mathbf{P}(\forall x)\neg Px$. $\mathbf{P}(\forall x)\neg Px$ is equivalent to $\neg\mathbf{O}(\exists x)Px$ and so $\mathcal{M}, w_1 \vDash \neg\mathbf{O}(\exists x)Px$. By the negation clause it follows that $\mathcal{M}, w_1 \nvDash \mathbf{O}(\exists x)Px$. ■

Fact 13. *The converse Ghilardi formula is not valid when using varying domain models with the standard deontic operator clause or the Van Benthem clause.*

Proof. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = d(w_2) = \{o_1, o_2\}$
5. $a(P, w_1) = \{o_1\}$ and $a(P, w_2) = \{o_2\}$

Now we can check whether the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash \mathbf{O}(\exists x)Px$ and $\mathcal{M}, w_1 \nvDash (\exists x)\mathbf{O}Px$. To check whether $\mathcal{M}, w_1 \vDash \mathbf{O}(\exists x)Px$ we have to consider all worlds w' for which Rw_1w' and see whether $\mathcal{M}, w' \vDash (\exists x)Px$ for all such w'. We can see that there are two worlds we have to check, namely w_1 and w_2.

To ascertain whether $\mathcal{M}, w_1 \vDash (\exists x)Px$ holds we have to look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash Px$ holds in one of these $\mathcal{M}'$. Consider the model $\mathcal{M}'$ in which $a(x) = o_1$, we can see that in $\mathcal{M}'$ it holds that $a(x) \in a(P, w_1)$ and we know that the assignment function a does not change with respect to the extension of P in w_1 in all the models $\mathcal{M}'$. From this it follows that there is a model $\mathcal{M}'$ for which it is true that $\mathcal{M}', w_1 \vDash Px$. Because of this fact we know that $\mathcal{M}, w_1 \vDash (\exists x)Px$.

To see that $\mathcal{M}, w_2 \vDash (\exists x)Px$, consider the model $\mathcal{M}'$ in which $a(x) = o_2$, we can see that in $\mathcal{M}'$ it holds that $a(x) \in a(P, w_2)$ and we know that the assignment function a does not change with respect to the extension of P in w_2 in all the models $\mathcal{M}'$. From this it follows that there is a model $\mathcal{M}'$ for which it is true that $\mathcal{M}', w_2 \vDash Px$. Because of this fact we know that $\mathcal{M}, w_2 \vDash (\exists x)Px$.

We have checked every w' for which Rw_1w' because there were only two to check and so we can conclude that $\mathcal{M}, w_1 \vDash \mathbf{O}(\exists x)Px$.

Now we have to prove that $\mathcal{M}, w_1 \nvDash (\exists x)\mathbf{O}Px$. I will do this by showing that its negation is true in $\mathcal{M}$, namely that $\mathcal{M}, w_1 \vDash \neg(\exists x)\mathbf{O}Px$ which is equivalent to $\mathcal{M}, w_1 \vDash (\forall x)\mathbf{P}\neg Px$. To check whether $\mathcal{M}, w_1 \vDash (\forall x)\mathbf{P}\neg Px$ we have to take a look at all $\mathcal{M}' = \langle W, R, D, d, a'\rangle$, where a' differs at most from a concerning the value it assigns from D_{w_1} to x, and check if $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ holds for all of these $\mathcal{M}'$. To check whether $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ holds for all of these models we have to check for all these models $\mathcal{M}'$ whether there is a world w' for which Rw_1w' and $\mathcal{M}', w' \vDash \neg Px$. In all these models $\mathcal{M}'$ there are two worlds w' to check for which Rw_1w', namely w_1 and w_2. We know that there will either be models $\mathcal{M}'$ in which $a(x) = o_2$ or models $\mathcal{M}'$ in which $a(x) = o_1$.

In the models where $a(x) = o_2$ it will be the case that $a(x) \notin a(P, w_1)$ and so in these models $\mathcal{M}'$ we will have $\mathcal{M}', w_1 \nvDash Px$ and by the negation clause we have for every such $\mathcal{M}'$ that $\mathcal{M}', w_1 \vDash \neg Px$. In the models where $a(x) = o_1$ it will be the case that $a(x) \notin a(P, w_2)$ and so in these models $\mathcal{M}'$ we will have $\mathcal{M}', w_2 \nvDash Px$ and by the negation clause we have for every such $\mathcal{M}'$ that $\mathcal{M}', w_2 \vDash \neg Px$.

This result establishes that in all $\mathcal{M}'$ we have $\mathcal{M}', w_1 \vDash \mathbf{P}\neg Px$ and subsequently that $\mathcal{M}, w_1 \vDash (\forall x)\mathbf{P}\neg Px$. Because of the equivalence between $(\forall x)\mathbf{P}\neg Px$ and $\neg(\exists x)\mathbf{O}Px$ we know that $\mathcal{M}, w_1 \vDash \neg(\exists x)\mathbf{O}Px$ and by the negation clause that $\mathcal{M}, w_1 \nvDash (\exists x)\mathbf{O}Px$. ■

It is proven by Giovanna Corsi in Corsi (2002*b*) that $\mathbf{QFD}$[12] is sound and complete with respect to varying domain models and the standard deontic operator clause. The previous results show us that if we use the Van Benthem clause and varying domain models $\mathbf{CBF}$ is valid while $\mathbf{GF}$ is not valid. This is strange because we learned in section 3.3.3 that $\mathbf{GF}$ is a theorem of $\mathbf{QFD} + \mathbf{CBF}$. What this shows is that $\mathbf{QFD} + \mathbf{CBF}$ is not sound with respect to varying domain semantics and the Van Benthem clause. If it were sound it would have to follow that $\mathbf{GF}$ is valid because it is provable within $\mathbf{QFD} + \mathbf{CBF}$ and soundness guarantees that every provable formula is valid. There has to be a step in the proof of $\mathbf{GF}$ that is no longer available when we adopt the Van Benthem clause. I will restate the proof here for convenience:

Proof. Some minor steps have been omitted:

1. $(\forall x)(Px \supset (\exists x)Px)$ (Theorem instance)
2. $\mathbf{O}(\forall x)(Px \supset (\exists x)Px)$ (1; $\mathbf{RND}$)
3. $(\forall x)\mathbf{O}(Px \supset (\exists x)Px)$ (2; $\mathbf{CBF}$)
4. $(\forall x)(\mathbf{O}Px \supset \mathbf{O}(\exists x)Px)$ (3; $\mathbf{KD}$)
5. $(\forall x)(\mathbf{O}Px \supset \mathbf{O}(\exists x)Px) \supset ((\exists x)\mathbf{O}Px \supset \mathbf{O}(\exists x)Px))$ (Theorem instance)
6. $(\exists x)\mathbf{O}Px \supset \mathbf{O}(\exists x)Px)$ (4, 5; $\mathbf{MP}$)

Quite surprisingly the $\mathbf{KD}$ axiom scheme is the culprit: $\mathbf{KD}$ is no longer valid if we adopt the Van Benthem clause.

Fact 14. *When using varying domain models with a Van Benthem clause the axiom schema* $\mathbf{KD}$ *is not valid.*

Proof. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_2\}$ and $d(w_2) = \{o_1, o_2\}$

[12] Corsi (2002*b*) proves this for $\mathbf{Q}^{\circ}_{=}.\mathbf{K}$ which is axiomatically defined the same as $\mathbf{QFD}$ without the $\mathbf{DD}$ axiom scheme.

5. $a(P, w_1) = \emptyset$ and $a(P, w_2) = \{o_1, o_2\}$ and $a(a) = o_1$ and $a(b) = o_2$

I will now show that the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \vDash \mathbf{O}(Pa \supset Pb)$ and $\mathcal{M}, w_1 \nvDash \mathbf{O}Pa \supset \mathbf{O}Pb$. To check whether $\mathcal{M}, w_1 \vDash \mathbf{O}(Pa \supset Pb)$ holds we have to check in all worlds w' for which Rw_1w' and $a(a) \in D_{w'}$ and $a(b) \in D_{w'}$ whether $\mathcal{M}, w' \vDash Pa \supset Pb$ holds. We can see that there is only one such w' namely w_2. We can also see that at w_2 we have $\mathcal{M}, w_2 \vDash Pb$ because $a(b) \in a(P, w_2)$. This establishes that $\mathcal{M}, w_2 \vDash Pa \supset Pb$ holds and because we have checked every accessible w' with the right criteria it follows that $\mathcal{M}, w_1 \vDash \mathbf{O}(Pa \supset Pb)$ is true.

Because of $\mathcal{M}, w_1 \nvDash \mathbf{O}Pa \supset \mathbf{O}Pb$ we know by the negation clause that $\mathcal{M}, w_1 \vDash \neg(\mathbf{O}Pa \supset \mathbf{O}Pb)$ which is equivalent to $\mathcal{M}, w_1 \vDash \mathbf{O}Pa \wedge \neg\mathbf{O}Pb$. Thus we will show that $\mathcal{M}, w_1 \nvDash \mathbf{O}Pa \supset \mathbf{O}Pb$ is the case by showing that $\mathcal{M}, w_1 \vDash \mathbf{O}Pa \wedge \neg\mathbf{O}Pb$ holds. Let us first check whether $\mathcal{M}, w_1 \vDash \mathbf{O}Pa$. To ascertain whether $\mathcal{M}, w_1 \vDash \mathbf{O}Pa$ holds we have to check in all worlds w' for which Rw_1w' and $a(a) \in D_{w'}$. We can see that there is only one such w' that is eligible namely w_2. At w_2 we have $\mathcal{M}, w_2 \vDash Pa$ because $a(a) \in a(P, w_2)$. To ascertain whether $\mathcal{M}, w_1 \vDash \neg\mathbf{O}Pb$ holds we have to check whether $\mathcal{M}, w_1 \nvDash \mathbf{O}Pb$. $\mathcal{M}, w_1 \nvDash \mathbf{O}Pb$ would mean that there is a world w' for which Rw_1w' and $a(b) \in D_{w'}$ at which it holds that $\mathcal{M}, w' \nvDash Pb$. We can see that there is such a world w' namely w_1. From $\mathcal{M}, w_1 \vDash \mathbf{O}Pa$ and $\mathcal{M}, w_1 \vDash \neg\mathbf{O}Pb$ we can conclude by the conjunction clause that $\mathcal{M}, w_1 \vDash \mathbf{O}Pa \wedge \neg\mathbf{O}Pb$. This in turn gives us $\mathcal{M}, w_1 \vDash \neg(\mathbf{O}Pa \supset \mathbf{O}Pb)$ and so this concludes the proof with $\mathcal{M}, w_1 \nvDash \mathbf{O}Pa \supset \mathbf{O}Pb$. ■

This result establishes that any axiom system sound for varying domain models and the Van Benthem clause (as I have defined them) will be a non-normal modal logic because **K** or its deontic counterpart **KD** can not be added as an axiom without producing an unsound axiomatisation. Unfortunately, to my knowledge, there is as of yet no axiomatisation of varying domain models and the Van Benthem clause and I have not been able to produce it myself. Consequently, there is also no soundness and completeness proof.

3.4.2 Increasing domain models

I will now show that if we impose increasing domain frames, we will still have models that make the Barcan formula and the converse Ghilardi formula false but the converse Barcan formula and the Ghilardi formula will be valid.

Fact 15. *The Barcan formula is not valid on increasing domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the Barcan formula is not valid when using varying domain models because that proof only relies on increasing domain models. ■

Fact 16. *The converse Ghilardi formula is not valid on increasing domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Ghilardi formula is not valid when using varying domain models because that proof only relies on constant domain frames. ■

Fact 17. *The converse Barcan formula is valid on increasing domain frames with the standard deontic operator clause.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \vDash \mathbf{O}(\forall x)Px$ and $\mathcal{M}, w \nvDash (\forall x)\mathbf{O}Px$. Because of $\mathcal{M}, w \vDash \mathbf{O}(\forall x)Px$ we know that for every w' for which Rww' that $\mathcal{M}, w' \vDash (\forall x)Px$. From this we can infer that for all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from $D_{w'}$ it holds that $\mathcal{M}', w' \vDash Px$.

Because of $\mathcal{M}, w \nvDash (\forall x)\mathbf{O}Px$ we know by the negation clause that $\mathcal{M}, w \vDash \neg(\forall x)\mathbf{O}Px$ which is equivalent to $\mathcal{M}, w \vDash (\exists x)\mathbf{P}\neg Px$. Because of this we know that there is a model $\mathcal{M}'$ for which the assignment function a' differs at most from a with respect to the value it assigns to x from D_w in which $\mathcal{M}', w \vDash \mathbf{P}\neg Px$. From this we can infer that in this model $\mathcal{M}'$ there is a world w' for which Rww' and $\mathcal{M}', w' \vDash \neg Px$.

Because we only look at models based on an increasing domain frame we know that for all w and all w' for which Rww' it holds that $D_w \subseteq D_{w'}$. By the first part of the proof we know that for all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from $D_{w'}$ it holds that $\mathcal{M}', w' \vDash Px$. Hence, for every a' in every such $\mathcal{M}'$ it holds that $a'(x) \in a'(P, w')$ where $a'(x) \in D_{w'}$. By the second part of the proof we know that there is a model $\mathcal{M}'$ and a world w' in it for which $\mathcal{M}', w' \vDash \neg Px$ which tells us that in this $\mathcal{M}'$ it is the case that $a'(x) \notin a'(P, w')$ where $a'(x) \in D_w$. By the property of increasing domains we know that this $a'(x)$ must also be in $D_{w'}$ and so $a'(x) \in D_{w'}$ but for every $a'(x) \in D_{w'}$ it holds that $a'(x) \in a'(P, w')$ which leaves us with a contradiction. ■

Fact 18. *The converse Barcan formula is valid on increasing domain frames with the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Barcan formula is valid when using varying domain models because increasing domain models are a subset of varying domain models. ■

Fact 19. *The Ghilardi formula is valid on increasing domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \vDash (\exists x)\mathbf{O}Px$ and $\mathcal{M}, w \nvDash \mathbf{O}(\exists x)Px$. Because of $\mathcal{M}, w \vDash (\exists x)\mathbf{O}Px$ we know that among all $\mathcal{M}'$ in which the assignment function a' differs at most concerning the value it assigns to x from D_w we have at least one $\mathcal{M}'$ such that $\mathcal{M}', w \vDash \mathbf{O}Px$. In this $\mathcal{M}'$ we know that for each w' for which Rww' we have $\mathcal{M}', w' \vDash Px$.

Because of $\mathcal{M}, w \nvDash \mathbf{O}(\exists x)Px$ we know by the negation clause that $\mathcal{M}, w \vDash \neg\mathbf{O}(\exists x)Px$ which is equivalent to $\mathcal{M}, w \vDash \mathbf{P}(\forall x)\neg Px$. Because of this we know that in $\mathcal{M}$ we have a world w' for which Rww' and $\mathcal{M}, w' \vDash (\forall x)\neg Px$. Because of $\mathcal{M}, w' \vDash (\forall x)\neg Px$ we know that in all $\mathcal{M}'$ for which the assignment function a' differs at most from a with respect to the value it assigns to x from $D_{w'}$ we have $\mathcal{M}', w' \vDash \neg Px$.

Because we only look at models based on an increasing domain frame we know that for all w and all w' for which Rww' it holds that $D_w \subseteq D_{w'}$. By the first part of the proof we know that there is a $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from D_w it holds that $\mathcal{M}', w' \vDash Px$ for every w'. Hence, for this a' in this $\mathcal{M}'$ it holds that $a'(x) \in a'(P, w')$ for all w' and $a'(x) \in D_w$. Because of increasing domains we also know that $a'(x) \in D_{w'}$. However, by the second part of the proof we know that for every $\mathcal{M}'$ there is a world w' in it for which $\mathcal{M}', w' \vDash \neg Px$ which thus tells us that in all these $\mathcal{M}'$ it is the case that there is a w' and $a'(x) \notin a'(P, w')$ where $a'(x) \in D_{w'}$. This leaves us with a contradiction. ■

Fact 20. *If we impose increasing domain frames and use a Van Benthem clause the axiom schema* **KD** *is not valid.*

Proof. This fact follows immediately from the proof given earlier that shows that the axiom schema **KD** is not valid when using varying domain models and a van Benthem clause because that proof only relies on models based on an increasing domain frame. ■

It is proven by Giovanna Corsi in Corsi (2002*b*) that $\mathbf{QFD} + \mathbf{CBF}$ is sound and complete with respect to increasing domain frames and the standard deontic operator clause. Just as before, $\mathbf{QFD} + \mathbf{CBF}$ is not sound with respect to increasing domain frames and the Van Benthem clause because of the invalidity of **KD**.

3.4.3 Decreasing domain models

I will now show that if we impose decreasing domain frames, we will have models that make the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula false but the Barcan formula will be valid. Moreover, **KD** will be invalid when adopting a Van Benthem clause.

Fact 21. *The Barcan formula is valid on increasing domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \vDash (\forall x)\mathbf{O}Px$ and $\mathcal{M}, w \nvDash \mathbf{O}(\forall x)Px$. Because of $\mathcal{M}, w \vDash (\forall x)\mathbf{O}Px$ we know that for all $\mathcal{M}'$ in which the assignment function a' differs at most concerning the value it assigns to x from D_w we have $\mathcal{M}', w \vDash \mathbf{O}Px$. In all these $\mathcal{M}'$ we know that for each w' for which Rww' we have $\mathcal{M}', w' \vDash Px$.

Because of $\mathcal{M}, w \nvDash \mathbf{O}(\forall x)Px$ we know by the negation clause that $\mathcal{M}, w \vDash \neg\mathbf{O}(\forall x)Px$ which is equivalent to $\mathcal{M}, w \vDash \mathbf{P}(\exists x)\neg Px$. Because of this we know that in $\mathcal{M}$ we have a world w' for which Rww' and $\mathcal{M}, w' \vDash (\exists x)\neg Px$. Because of $\mathcal{M}, w' \vDash (\exists x)\neg Px$ we know that among all $\mathcal{M}'$ for which the assignment function a' differs at most from a with respect to the value it assigns to x from $D_{w'}$ we have a model $\mathcal{M}'$ such that $\mathcal{M}', w' \vDash \neg Px$.

Because we only look at models based on a decreasing domain frame we know that for all w and all w' for which Rww' it holds that $D_{w'} \subseteq D_w$. By the first part of the proof we know that for all $\mathcal{M}'$ in which the assignment function a' differs at most from a concerning the value it assigns to x from D_w it holds that $\mathcal{M}', w' \vDash Px$ for every w'. Hence, for the a' in all these $\mathcal{M}'$ it holds that $a'(x) \in a'(P, w')$ for all w' and $a'(x) \in D_w$. By the second part of the proof we know that there is a $\mathcal{M}'$ and a world w' in it for which $\mathcal{M}', w' \vDash \neg Px$ which tells us that in this $\mathcal{M}'$ it is the case that there is a w' and $a'(x) \notin a'(P, w')$ where $a'(x) \in D_{w'}$. Because of decreasing domains we also know that for this a' it is true that $a'(x) \in D_w$ but for every $a'(x) \in D_w$ it holds that $a'(x) \in a'(P, w')$ which leaves us with a contradiction. ■

Fact 22. *The converse Barcan formula is not valid on decreasing domain frames with the standard deontic operator clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Barcan formula is not valid when using varying domain models because that proof only relies on models based on a decreasing domain frame. ■

Fact 23. *The converse Barcan formula is valid on decreasing domain frames with the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Barcan formula is valid when using varying domain models because decreasing domain models are a subset of varying domain models. ■

Fact 24. *The Ghilardi formula is not valid on decreasing domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the Ghilardi formula is not valid when using varying domain models with the standard deontic operator clause or the Van Benthem clause because that proof only relies on models based on a decreasing domain frame. ■

Fact 25. *The converse Ghilardi formula is not valid on decreasing domain models with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Ghilardi formula is not valid when using varying domain models with the standard deontic operator clause or the Van Benthem clause because that proof only relies on models based on a constant domain frame. ■

Fact 26. *If we impose decreasing domain frames and use a Van Benthem clause the axiom schema* **KD** *is not valid.*

Proof. Consider any arbitrary varying domain model $\mathcal{M}$ such that:

1. $W = \{w_1, w_2\}$
2. $R = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$
3. $D = \{o_1, o_2\}$
4. $d(w_1) = \{o_1, o_2\}$ and $d(w_2) = \{o_1\}$
5. $a(P, w_1) = \{o_1, o_2\}$ and $a(P, w_2) = \emptyset$ and $a(a) = o_2$ and $a(b) = o_1$

I will now show that the model $\mathcal{M}$ is such that $\mathcal{M}, w_1 \models \mathbf{O}(Pa \supset Pb)$ and $\mathcal{M}, w_1 \not\models \mathbf{O}Pa \supset \mathbf{O}Pb$. To check whether $\mathcal{M}, w_1 \models \mathbf{O}(Pa \supset Pb)$ holds we have to check in all worlds w' for which Rw_1w' and $a(a) \in D_{w'}$ and $a(b) \in D_{w'}$ whether $\mathcal{M}, w' \models Pa \supset Pb$ holds. We can see that there is only one such w' namely w_1. We can also see that at w_1 we have $\mathcal{M}, w_1 \models Pb$ because $a(b) \in a(P, w_1)$. This establishes that $\mathcal{M}, w_1 \models Pa \supset Pb$ holds and because we have checked every accessible w' with the right criteria it follows that $\mathcal{M}, w_1 \models \mathbf{O}(Pa \supset Pb)$ is true.

Because of $\mathcal{M}, w_1 \not\models \mathbf{O}Pa \supset \mathbf{O}Pb$ we know by the negation clause that $\mathcal{M}, w_1 \models \neg(\mathbf{O}Pa \supset \mathbf{O}Pb)$ which is equivalent to showing that $\mathcal{M}, w_1 \models \mathbf{O}Pa \wedge \neg\mathbf{O}Pb$. Let us first check whether $\mathcal{M}, w_1 \models \mathbf{O}Pa$. To ascertain whether $\mathcal{M}, w_1 \models \mathbf{O}Pa$ holds we have to check in all worlds w' for which Rw_1w' and $a(a) \in D_{w'}$. We can see that there is only one such w' that is eligible namely w_1. At w_1 we have $\mathcal{M}, w_1 \models Pa$ because $a(a) \in a(P, w_1)$. To ascertain whether $\mathcal{M}, w_1 \models \neg\mathbf{O}Pb$ holds we have to check whether $\mathcal{M}, w_1 \not\models \mathbf{O}Pb$. $\mathcal{M}, w_1 \not\models \mathbf{O}Pb$ would mean that there is a world w' for which Rw_1w' and $a(b) \in D_{w'}$ at which it holds that $\mathcal{M}, w' \not\models Pb$. We can see that there is such a world w' namely w_2. From $\mathcal{M}, w_1 \models \mathbf{O}Pa$ and $\mathcal{M}, w_1 \models \neg\mathbf{O}Pb$ we can conclude by the conjunction clause that $\mathcal{M}, w_1 \models \mathbf{O}Pa \wedge \neg\mathbf{O}Pb$. This in turn gives us $\mathcal{M}, w_1 \models \neg(\mathbf{O}Pa \supset \mathbf{O}Pb)$ and so this concludes the proof with $\mathcal{M}, w_1 \not\models \mathbf{O}Pa \supset \mathbf{O}Pb$. ■

It is proven by Giovanna Corsi in Corsi (2002*b*) that $\mathbf{QFD} + \mathbf{BF}$ is sound and complete with respect to decreasing domain frames and the standard deontic operator clause. $\mathbf{QFD}$ and $\mathbf{QFD} + \mathbf{BF}$ are not sound with respect to decreasing domain frames and the Van Benthem clause because of the invalidity of $\mathbf{KD}$.

3.4.4 Constant domain models

I will now show that if we impose constant domain frames, the Barcan formula, the converse Barcan formula and the Ghilardi formula will be valid but we still have models that invalidate the converse Ghilardi formula.

Fact 27. *The Barcan formula is valid on constant domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the Barcan formula is valid when using decreasing domain models with the standard deontic operator clause or the Van Benthem clause because constant domain models are a subset of decreasing domain models. ■

Fact 28. *The converse Barcan formula is valid on constant domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Barcan formula is valid when using increasing domain models with the standard deontic operator clause or the Van Benthem clause because constant domain models are a subset of increasing domain models. ■

Fact 29. *The converse Barcan formula is valid on constant domain frames and the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Barcan formula is valid when using varying domain models because constant domain models are a subset of varying domain models. ■

Fact 30. *The Ghilardi formula is valid on constant domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the Ghilardi formula is valid when using increasing domain models with the standard deontic operator clause or the Van Benthem clause because constant domain models are a subset of increasing domain models. ■

Fact 31. *The converse Ghilardi formula is not valid on constant domain frames with the standard deontic operator clause or the Van Benthem clause.*

Proof. This fact follows immediately from the proof given earlier that shows that the converse Ghilardi formula is not valid when using varying domain models with the standard deontic operator clause or the Van Benthem clause because that proof only relies on constant domain frames. ■

Fact 32. *If we impose constant domain frames and use a Van Benthem clause the axiom schema* $\mathbf{KD}$ *valid.*

Proof. The proof is by *reductio ad absurdum*. Suppose that $\mathcal{M}, w \models \mathbf{O}(A[\alpha] \supset B[\beta])$ and $\mathcal{M}, w \not\models \mathbf{O}A[\alpha] \supset \mathbf{O}B[\beta]$. $\mathcal{M}, w \models \mathbf{O}(A[\alpha] \supset B[\beta])$ guarantees that at all worlds w' such that Rww' and $a(\alpha) \in D_{w'}$ and $a(\beta) \in D_{w'}$ we have $\mathcal{M}, w \models A[\alpha] \supset B[\beta]$. It follows that in $\mathcal{M}$ we have for every such w' that either $\mathcal{M}, w' \not\models A[\alpha]$ or $\mathcal{M}, w' \models B[\beta]$.

$\mathcal{M}, w \not\models \mathbf{O}A[\alpha] \supset \mathbf{O}B[\beta]$ guarantees $\mathcal{M}, w \models \mathbf{O}A[\alpha] \wedge \neg\mathbf{O}B[\beta]$. This establishes that $\mathcal{M}, w \models \mathbf{O}A[\alpha]$ and $\mathcal{M}, w \not\models \mathbf{O}B[\beta]$. $\mathcal{M}, w \models \mathbf{O}A[\alpha]$ says that in all worlds w' for which $a(\alpha) \in D_{w'}$ we have $\mathcal{M}, w \models A[\alpha]$. $\mathcal{M}, w \not\models \mathbf{O}B[\beta]$ guarantees that there is a world w' for which Rww' and $a(\beta) \in D_{w'}$ such that $\mathcal{M}, w' \not\models B[\beta]$.

Constant domain frames ensure that every accessible world has the same domain. This means that every world that is accessible to evaluate the truth of $\mathcal{M}, w \models \mathbf{O}(A[\alpha] \supset B[\beta])$ will also be accessible to evaluate the truth of $\mathcal{M}, w \models \mathbf{O}A[\alpha]$ or $\mathcal{M}, w \not\models \mathbf{O}B[\beta]$ and vice versa. It follows that as soon as one world is accessible to evaluate the truth of a modal formula containing a free variable or constant at a world, the truth of every modal formula containing free variables or constants at that world will be evaluated with respect to the same accessible worlds.[13] We knew that in $\mathcal{M}$ we have for every accessible w' that either $\mathcal{M}, w' \not\models A[\alpha]$ or $\mathcal{M}, w' \models B[\beta]$. However, we also know that in all accessible worlds w' we have $\mathcal{M}, w \models A[\alpha]$ and among these accessible worlds there must be a world w' such that $\mathcal{M}, w' \not\models B[\beta]$. This leaves us with a contradiction. ■

It is proven by Giovanna Corsi in Corsi (2002*b*) that $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$ is sound and complete with respect to constant domain frames and the standard deontic operator clause. When using constant domain frames and a Van Benthem clause it is not clear whether $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$ is sound with respect to it, we do not

[13] There is a minor caveat here, if the modal formula contains a free variable or constant that denotes a person that does not exist in any accessible world then it will not be evaluated with respect to the same accessible worlds as the modal formulas for which this is not the case. In fact, in such a case the formula will turn out vacuously true due to there being no world accessible for evaluation. It is interesting to observe here that modal formulas containing free variables or constants, when using a Van Benthem clause, seem to induce their own accessibility relation. Perhaps there is an interesting link here with respect to multi-relational semantics that use multiple accessibility relations.

have the same problem as before with the **KD** axiom scheme but there is no soundness or completeness result available.

3.4.5 Summary

In summary, we have before us 5 different possible logics and some ways in which to make their axioms valid. The following table summarises the relation between the frame conditions and the validity or invalidity of the discussed formulas when adopting the standard deontic operator clause:

Axioms	No condition	Increasing domains	Decreasing domains	Constant domains
Barcan formula	Invalid	Invalid	Valid	Valid
converse Barcan formula	Invalid	Valid	Invalid	Valid
Ghilardi formula	Invalid	Valid	Invalid	Valid
converse Ghilardi formula	Invalid	Invalid	Invalid	Invalid
KD	Valid	Valid	Valid	Valid

The table below shows the relation between the frame conditions and the validity or invalidity of the formulas when adopting the Van Benthem clause:

Axioms	No condition	Increasing domains	Decreasing domains	Constant domains
Barcan formula	Invalid	Invalid	Valid	Valid
converse Barcan formula	Valid	Valid	Valid	Valid
Ghilardi formula	Invalid	Valid	Invalid	Valid
converse Ghilardi formula	Invalid	Invalid	Invalid	Invalid
KD	Invalid	Invalid	Invalid	Valid

These two tables show clearly that the difference between the standard deontic operator clause and the Van Benthem clause is located in the status of the converse Barcan formula and the $\mathbf{KD}$ axiom scheme. $\mathbf{CBF}$ is always valid when adopting the Van Benthem clause while it is not always valid when using the standard deontic operator clause. $\mathbf{KD}$ is always valid when using the standard deontic operator clause while it is only valid when using constant domain frames if we use a Van Benthem clause. These tables also show that there are two logics $\mathbf{QFD} + \mathbf{GF}$ and $\mathbf{QFD} + \mathbf{BF} + \mathbf{GF}$ that are impossible to match semantically with the options laid out. It is impossible to match $\mathbf{QFD} + \mathbf{GF}$ and $\mathbf{QFD} + \mathbf{BF} + \mathbf{GF}$ because every option that validates $\mathbf{GF}$ also validates $\mathbf{CBF}$.

The first option is to adopt $\mathbf{QFD}$. If we take this option we need to use varying domain models and the standard deontic operator. Only in this way will the Barcan formula, the converse Barcan formula, the Ghilardi formula, the converse Ghilardi formula all turn out invalid. What this means is that if we want to argue for the adoption of varying domain models we will have to give convincing examples that show that each of these formulas, when given an appropriate interpretation, can be made false.

The second option is to adopt $\mathbf{QFD} + \mathbf{CBF}$. If we take this option we need to impose increasing domain frames and use the standard deontic operator clause. Only then will the Barcan formula and the converse Ghilardi formula turn out invalid but the converse Barcan formula, the Ghilardi formula and $\mathbf{KD}$ will be valid. If we want to argue for the adoption of this logic, we will have to give convincing examples that show that the converse Barcan formula and Ghilardi formula will always be true in contradistinction to the Barcan formula and the converse Ghilardi formula which can turn out false.

The third option is to adopt $\mathbf{QFD} + \mathbf{BF}$. If we take this option we need to impose decreasing domain frames and use a standard deontic operator clause. Only then will converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula turn out invalid but the Barcan formula will be valid. If we want to argue for the adoption of this logic, we will have to give convincing examples that show that the Barcan formula will always be true in contradistinction to the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula.

The fourth option is to adopt $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$. If we take this option we need to impose constant domain frames and use the standard deontic operator clause. Only then will the the converse Ghilardi formula turn out invalid but the Barcan formula, Ghilardi formula and converse Barcan formula valid. If we want to argue for the adoption of this logic, we will have to give convincing examples that show that the Barcan formula, the converse Barcan formula and the Ghilardi formula will always be true in contradistinction to the converse Ghilardi formula.

The fifth option is to adopt varying domain models and a Van Benthem clause. This will ensure that the converse Barcan formula is valid while the Barcan formula, the Ghilardi formula and the converse Ghilardi formula is not. We will also have to give up $\mathbf{KD}$ and produce a logic that is sound and complete with respect to this semantics.

The sixth option is to impose increasing domain frames and use a Van Benthem clause. This will ensure that the converse Barcan formula and Ghilardi formula is valid while the Barcan formula and the converse Ghilardi formula is not. We will also have to give up $\mathbf{KD}$ and produce a logic that is sound and complete with respect to this semantics.

The seventh option is to impose decreasing domain frames and use a Van Benthem clause. This will ensure that the Barcan formula is valid while the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula is not. We will also have to give up $\mathbf{KD}$ and produce a logic that is sound and complete with respect to this semantics.

The eighth option is to impose constant domain frames and use a Van Benthem clause. This will ensure that the Barcan formula, the converse Barcan formula and the Ghilardi formula is valid while the converse Ghilardi formula is not. We do not have to give up $\mathbf{KD}$ but we do need to show either that $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$ is sound and complete with respect to this semantics or produce a new logic that is sound and complete.

Varying domain semantics and constant domain semantics are, in the literature on quantified modal logic that focuses on alethic modalities, sometimes linked to the metaphysical positions known as *actualism* and *possibilism*. Because varying domain semantics conceives of people as existing at particular worlds and not existing at other worlds it is tied to actualism: the position that only that which is actual exists. In constant domain semantics we have no such variation across worlds and so the only way to make sense of this seems to be that the domain not only comprises actual individuals but also possible individuals and so in some sense, *possibilia* exist (Tomberlin, 1996; Menzel, 2016). Some have even maintained that one cannot take possible world semantics seriously while at the same time being an actualist (Menzel, 1990).

However, none of these metaphysical worries will be addressed here for two reasons. The first and primary reason is that metaphysical arguments only carry much weight if one intends to use the logic to bear on metaphysical issues. If one is concerned with human reasoning, however, and merely conceives of logics as tools to help us understand and improve our reasoning it is less clear why it would matter whether our semantic constructs reflect metaphysical reality (Gamut, 1991). The second reason is that, even if we would take metaphysical considerations into account, it is a mystery to me how we could, on metaphysical grounds, make sense imposing increasing or decreasing domain frames. For example, if we adopt actualism, it wouldn't make sense to impose increasing domain frames or decreasing domain frames. Because, clearly, people die all the time and people come into existence all the time and so we need varying domains. If we would give serious weight to metaphysical arguments, it seems like we would lose some of the options before us. Because I want to preserve all the options and do not think we should give much weight to metaphysical arguments when discussing human reasoning I will not take metaphysical considerations into account when deciding on what option to pick.

Now that I have explicated some of the logical machinery, presented some of our options, and proven some of the properties of these options, we will, in the chapter ahead, take a stab at interpreting the relation between the quantifiers and the deontic operators.

4. THE RELATION BETWEEN THE QUANTIFIERS AND THE DEONTIC OPERATORS

In this chapter, we will take a look at the relation between the quantifiers and the deontic operators. There are eight formulas that express basic interactions between the deontic operators and the quantifiers. These are the Barcan formula, the converse Barcan formula, the Ghilardi formula, the converse Ghilardi formula and their syntactic equivalents. In this chapter, we will investigate what it is precisely that these formulas semantically mean and how we might interpret them from a deontic point of view.

In section 4.1 I introduce a problem of interpretation in modal contexts commonly referred to as the distinction *de re/de dicto*. We will first take a look at some examples of this distinction in other modal contexts and then ask ourselves how this might translate into a deontic context. I will argue that there is a difference *de re/de dicto* in deontic contexts and that this difference can be leveraged to introduce a distinction between formulas representing person-specific norms and formulas representing person-non-specific norms.

In section 4.2 I will go into more detail on what the difference between a person-specific norm and a person-non-specific norm amounts to. I will also introduce absolute and conditional norms and show that by using a first-order language we can express conditional norms more faithfully and express different types of absolute norms.

In sections 4.3 to 4.6 I discuss the formulas expressing interactions between the quantifiers and the deontic operators and elaborate on whether they ought to be valid given the reading of these formulas introduced in sections 4.1 and 4.2. I will moreover argue that to uphold my reading we will need varying domain models and the Van Benthem clause.

4.1 The distinction de re/de dicto

The distinction *de re/de dicto* is about the modal operators having the ability to affect the interpretation of the various constituents of a sentence (Keshet and Schwarz, 2014). Quine offers the example: “The number of planets is necessarily odd.” in *Two dogmas of empiricism* (van Orman Quine, 1976). There are two ways in which to interpret this sentence.

The first way is by taking "the number of planets" as a description and applying the modal necessity operator to it. This is the *de dicto* reading, in symbols we would then formalise it as $\Box(\forall x)(Px \supset Ox)$ where $\Box$ means "necessarily", the domain is taken to be the natural numbers, P the predicate "the number of planets" and O the predicate "odd". In words: necessarily, for all natural numbers if the number matches the number of planets it is an odd number. This kind of reading seems to be false. The extension of "the number of planets" changes relative to the possible world from which it is evaluated and because it is conceivable that possible worlds contain an even number of planets it follows that "the number of planets" is not necessarily odd.

The second way to interpret it is by taking the modality operator to be applicable to the extension of "the number of planets", the thing itself, in this case, a number, and not the description. In symbols this becomes $\forall x(Px \supset \Box Ox)$. In Quine's time the number of planets was 9, if we then apply the modal operator to it, it says that the number 9 is necessarily odd, which, given our mathematical conventions, is true.

This phenomenon was first discussed by Aristotle in his *Prior Analytics* and *Sophistici elenchi* and about 1500 years later spelt out in more detail by Peter Abelard. Peter Abelard distinguished between an interpretation *de re*, about the thing, and an interpretation *de sensu*, about a linguistic statement (Kneale, 1966; Keshet and Schwarz, 2014). The distinction in its current terminology, however, is due to Thomas Aquinas.

The above example concerning the number of planets is an example of alethic nature. This phenomenon isn't restricted to alethic modality, however, but can also be found in doxastic and epistemic contexts. Quine (1956), for example, also gives this example: "Ralph believes that someone is a spy." and comments that "they may be unambiguously phrased respectively as 'There is someone whom Ralph believes to be a spy' and 'Ralph believes there are spies.'"(Quine, 1956, p. 178).

The Ralph example is straightforwardly convertible into an epistemic one. For instance, the sentence "Ralph knows that someone is a spy." might be taken as Ralph knowing that there are individuals who are spies but he doesn't know who they are and, alternatively, as Ralph knowing a particular individual that is a spy. If one alters the sentence just a bit "Ralph knows someone that is a spy", we would get a sentence that expresses the *de re* interpretation much more prominently. Nevertheless, the original sentence is much less clear-cut.

The natural question is now whether this distinction also applies to a deontic context. I think the answer is affirmative. Given the following two formulas:

De re $(\exists\alpha)\mathbf{O}A$

De dicto $\mathbf{O}(\exists\alpha)A$

how might they differ in meaning? If we apply a similar reading as before, we can interpret the *de re* formula as "there is someone for whom it is obligatory that A", and the *de dicto* formula as "it is obligatory that there is someone who A's". The *de re* formula thus applies the deontic operator to the extension of "someone". It uses the term "someone" as a way to refer to specific persons and then says of those persons that they have an obligation of some sort. In contrast, the *de dicto* formula expresses something about the content of the linguistic statement "there is someone who A's" and the fact that what that statement expresses is obligatory or ought to be the case. "Someone" is used here as a term that can apply to anyone.

For example, the formula $(\exists x)(\mathbf{O}Px \wedge (x = a))$ where a refers to John and P is a predicate for "doing the dishes" asserts that there is someone, John, who has to do the dishes. In this scenario, this obligation is person-specific because it is tied to John. In contrast, a formula such as $\mathbf{O}(\exists x)Px$ asserts that someone ought to do the dishes but there is no one specific person to which "someone" refers: it is not person-specific or person-non-specific.

On this account, formulas *de re* express something about a person or a set of persons and what it is that they are obligated to do while formulas *de dicto* say something about what is obligatory for "someone" or "everyone" without regard to whom that "someone" or "everyone" refers to. These different senses in which the word "someone" or "everyone" are used in normative discourse can be captured within a quantified deontic logic, or so I will argue.

4.2 Norms and their representation

> Norms can be conceived as directives that are issued by a norm-authority to direct the behavior of norm-subjects. As examples of norms, we can think of military commands, orders and permissions given by parents to children, traffic laws issued by a magistrate, etc. (Beirlaen, 2012, p. 2)

As Mathieu Beirlaen explains in his PhD thesis, norms are issued by norm-authorities to direct the behaviour of norm-subjects and they come in many shapes and forms. There are, for example, norms expressing obligations, norms expressing prohibitions and norms expressing permissions (Beirlaen, 2012). These are the kinds of norms that are easily representable by a propositional deontic language. If we want to express the fact that it is obligatory to pay taxes we can do so by using a propositional formula such as $\mathbf{O}p$ where p is interpreted as "paying taxes" and $\mathbf{O}$ expresses obligatoriness. The main point I will advance in this chapter is that by adopting a first-order deontic language we can represent more types of norms than if we would limit ourselves to a propositional language.

All formulas of the language are intended to be propositions and so formulas that express norms are to be interpreted as *norm-propositions*. In what follows I will not explicitly make a distinction between the *ought to do* versus the *ought to be* reading of deontic formulas (see Von Wright (1968)). Whether one interprets certain norms as agentive or non-agentive will not bear on the discussion that follows. I will also not clearly delineate the different kinds of normative sources but use a variety of examples. The reason is simply that I do not think that there are differences between these various kinds of normative sources that are relevant to the discussion in this chapter.

4.2.1 Person-specific norms and person-non-specific norms

A type of norm that becomes representable when adopting a first-order deontic language is what I will call a *person-specific norm*. A person-specific norm is a norm that can be conceived as a directive issued by a norm-authority that directs the behaviour of a specific norm-subject. For example, suppose that Mary is the norm-authority in question and she issues the command: "John, do the dishes!" then we can represent this person-specific norm by a formula such as $\mathbf{O}Da$ where 'D' stands for "doing the dishes" and 'a' refers to John. The fact that we can tie such an obligation to a specific person is something that we can do because of the availability of individual constants.

We can leverage the fact that the expression "Someone has to do the dishes" can be interpreted *de re* to say something about the person-specific norms in place. For example, the formula $(\exists x)\mathbf{O}Dx$ is such a formula and we can read it as saying that there exists at least one person with the person-specific obligation to do the dishes. If we know that John has to do the dishes ($\mathbf{O}Da$) and that John exists then we know that $(\exists x)\mathbf{O}Dx$ must be true. Conversely, if we know that $(\exists x)\mathbf{O}Dx$ is true there must be some specific person who has to do the dishes.

There are also person-non-specific norms. These are norms that do not direct the behaviour of a specific norm-subject but direct the behaviour of norm-subjects in general. On the *de dicto* reading: "Someone has to do the dishes" is interpreted as not being about what some specific person has to do. Instead, the norm applies in general to some set of norm-subjects. Before I elaborate on types of person-non-specific norms I will first introduce some other types of norms.

4.2.2 Absolute norms and conditional norms

In addition to person-specific and person-non-specific norms there are two types of norms that I deem especially relevant to the discussion in this chapter. These are *absolute norms* and *conditional norms*. Sadegh-Zadeh (2015) defines these two kinds of norms as follows:

> An absolute norm is an absolute deontic sentence, i.e., an obligation, permission, or prohibition without any precondition on which it depends. [...] an absolute norm binds independently of the factual circumstances, because no such circumstances are specified therein. By contrast, a conditional norm has a precondition such that when it is fulfilled, some action is obligatory, permitted, or forbidden. (Sadegh-Zadeh, 2015, p. 1005)

An example of an absolute norm is: “It is obligatory that you tell the truth.” while a conditional norm is, for example, “If you promise your patient to visit her, then it is obligatory that you do so.”(Sadegh-Zadeh, 2015, p. 1005).

A norm such as “John has to do the dishes” is an example of an absolute person-specific norm. The introduction of quantifiers, however, allows us to also represent absolute person-non-specific norms. I will show that we can use formulas *de dicto* to represent these kinds of absolute norms that exhibit either a universal quantification or an existential quantification. Absolute norms exhibiting such quantification will be dubbed *absolute quantified norms*

An example of an absolute quantified norm with universal quantification is the norm “It is obligatory that you tell the truth”. Absolute norms such as “It is obligatory that you tell the truth.” do not come with a precondition. We can capture this norm by a formula *de dicto* because it is not about what some specific person or set of persons are obligated to do. This is the type of norm that directs the behaviour of norm-subjects in general. In this case the quantification is intended to be universal: it is not just obligatory that someone tells the truth but that everyone tells the truth.[1] This *prima facie* leads me to conclude that $\mathbf{O}(\forall x)Tx$ is the most appropriate way to formalise it. That this is indeed the best formula to capture the intent of “It is obligatory that you tell the truth” will become more apparent when we assess how such a formula is semantically evaluated in sections 4.5 and 4.6.

An absolute norm such as “someone ought to do the dishes” when read *de dicto* is an example of an absolute quantified norm with an existential quantifier. In this case, $\mathbf{O}(\exists x)Dx$ can be used to represent the intent of the norm. Both $\mathbf{O}(\forall\alpha)A$ and $\mathbf{O}(\exists\alpha)A$ are formulas that can represent absolute quantified person-non-specific norms. To fully capture the person-non-specific intent of these types of norms we will have to adopt varying domains which I will make clear in the sections ahead.

Conditional norms are norms that have a precondition. If we were dealing with just the propositional level, the most plausible way to formalise the conditional norm: “If you promise your patient to visit her, then it is obligatory that you do so.” would probably be $p \supset \mathbf{O}q$ where the ‘p’ stands for “promising to visit your patient” and ‘q’ expresses that you visit your patient. There is some discussion on how to formalise conditional norms because $\mathbf{O}(A \supset B)$ is also seen as a plausible candidate. However, I follow Sadegh-Zadeh (2015) when they argue that:

> For two reasons, the alternative (265) [$\mathbf{O}(A \supset B)$] cannot be viewed as an adequate formalization of the commitment under discussion. First, its verbatim translation says “it is obligatory that if you promise your patient to visit her, you do so”. Thus, it deviates from the original “if you promise your patient to visit her, then it is obligatory that you do so”. Second, obviously it says that the conditional $A \supset B$ is an obligation. But it does not say what we

[1] The addition of a temporal operator would model the intention even closer. The intention behind such a statement is probably that it is obligatory that everyone tells the truth all the time. As indicated in the introduction, I do not consider multi-modal languages in this thesis but it is clear that a multi-modal language would in many cases allow a more fine-grained modelling of the intention behind some norm.

> are to do when the antecedent A is true. (Sadegh-Zadeh, 2015, pp. 1005-1006)

Because we are using a first-order language we can generalize the formula $p \supset \mathbf{O}q$ to one with a universal quantifier which gives us $(\forall x)(Px \supset \mathbf{O}Vx)$. This formula captures that everyone who promises to visit their patient is obligated to visit their patient which seems to be the intended conditional norm.

Conditional norms have an important role to play because they are able to induce person-specific norms: if John promised his patient to visit her then it is obligatory for John to do it. An important difference here is that the person-specific norms to which people are subject because they meet the condition of a conditional norm is dependent on what obtains in the actual world. There are also other ways to acquire a person-specific obligation. As said before, there might be some absolute norm directly saying that a particular person has to do something such as "John, do the dishes!". There might also be some absolute quantified norm such as "Everyone ought to tell the truth" in which case John also has the person-specific obligation to tell the truth in virtue of a person-non-specific norm that applies generally (see 4.6 for the discussion of this kind of situation). An important contrast is that person-specific norms incurred because of some absolute norm are not in the same way dependent on how the world actually is because they have no precondition in order to be applicable. This difference will become more clear later on and will be used to explain how varying domain models are able to capture the difference between a formula saying something about person-specific norms and formulas representing some form of absolute quantified norm.

Now that I have introduced some different types of norms let us apply these to a scenario that could plausibly happen. Suppose John has just recently accepted a job at a law firm. He finds himself at his first meeting among the rest of the people who have just been hired. They are all attentively listening to the speech of chairwomen Mary. Mary looks around, glances at the newcomers and declares: "As of tomorrow everyone has to be dressed in a suit and a tie". The newcomers internalise this rule and after the speech is over they all go to their offices. The next day Suzy, also a newcomer, turns up at John's office and informs her new colleague John that she could not attend the meeting and is unsure as to what the required dress code is. The following day Suzy makes an appearance in a suit and a tie despite the fact that women are required to wear a dress at this particular law firm. What went wrong?

What went wrong is that John recalled the rule "everyone has to be dressed in a suit and a tie" interpreted this as being an absolute quantified norm that applies generally and informed Suzy that in light of this rule she is required to be dressed in a suit and tie. However, chairwomen Mary did not mean to suggest that there is such an absolute quantified norm in place. She intended to say that the people at that meeting are required to wear a suit and tie. She came to that conclusion because she deduced it from the conditional norm that if you are male you have to wear a suit and tie and the fact that everyone at that meeting happened to be male. If John was aware of these possible ambiguities he would have perhaps thought twice about what Mary had said and realised that Mary's statement was ambiguous (or, alternatively, Mary would state the rules more carefully).

One of the central points of this chapter is the claim that in order to uphold the semantic distinction between the representation of an absolute quantified norm and the representation of the person-specific obligations of people we need to adopt varying domain models. The way in which to correctly evaluate the person-specific obligation of a person is to check in all the acceptable worlds in which that person exists what he or she there does. To capture the idea that the person has to exist in the acceptable worlds we will need the adoption of the Van Benthem clause. The way in which to evaluate whether an absolute quantified norm holds is to check not just what the people of the actual world do in the acceptable worlds but also what any arbitrary possible individual does for which we will need varying domains.

If we adopt constant domain models we are unable to look at individuals that do not exist in the actual world which collapses the distinction that we can make between what is true about people and their obligations and whether the norm-propositions modelling absolute quantified norms are true. For example, it can be contingently true that everyone has the obligation to A while it is at the same time true that there is no absolute quantified norm dictating that everyone has to A. When using constant domain models these two kinds of situations completely overlap because they demand the same from the people at the acceptable worlds.

In order to prevent this overlap and validly assess the truth of an absolute quantified norm we have to be able to abstract away from the actual world and the people in there. If we don't do this, the obligations that hold because of facts about the actual world and the people in there influence to a large extent what happens at the acceptable worlds. We need individuals at the acceptable worlds that are not already under some constraints because they are under the influence of a conditional norm or a direct person-specific norm. By allowing varying domains we can look not just at how the people of the actual world would behave in the acceptable words but also other possible individuals which reveals whether some norms apply unconditionally to every possible human or just to some subset thereof.

4.3 The converse Ghilardi formula and the converse Buridan formula

We will first discuss the converse Ghilardi and the converse Buridan formula. We have already established that these formulas are syntactically equivalent and not valid on any of the kinds of models introduced earlier. This leaves us with the problem of explaining why these formulas are problematic and why their invalidity is not just an unintended side effect of the way in which the models are defined. Hilpinen and McNamara (2013) provides us with some examples that highlight the problematic nature of the converse Ghilardi formula. The lifeboat example concerns the following situation:

> ...it may be obligatory that someone leave the lifeboat (else no one will be saved), but not that there is some one person such that she is obligated to leave, else there would be no need to draw straws to transform the first situation into one like the second, and it would also mean that at least someone in the boat could not go beyond the call by going overboard voluntarily, since s/he would be obligated to do so... (Hilpinen and McNamara, 2013, p. 53).

$\mathbf{O}(\exists x)Px$ expresses here that it ought to be that there is someone such that s/he leaves the lifeboat. In semantic terms this formula says that in each acceptable world there exists someone who leaves the lifeboat. It moreover doesn't matter whether it is the same person in each acceptable world. In other words, every possible world at which no existing person leaves the lifeboat will not be rendered acceptable from the point of view of the actual world. This kind of formula expresses on my reading an absolute quantified non-specific norm: it has no precondition and it is a formula *de dicto* which captures the person-non-specificity.

$(\exists x)\mathbf{O}Px$ says that there exists at least one person in the actual world for which it is true that that person leaves the lifeboat in each acceptable world. It forces a different "acceptability condition" on the possible worlds. Suddenly a possible world is no longer acceptable from the point of view of the actual world if it is not one of the specific persons in the domain of the actual world that makes the existential quantifier true that leaves the lifeboat in that possible world. In other words: it does not allow just any arbitrary person to do the act of leaving the lifeboat in order for a possible world to be rendered acceptable from the point of view of that formula. On my reading, this formula asserts the existence of at least one person who has the person-specific obligation to leave the lifeboat. However, in this example, surely the person-specificity of $(\exists x)\mathbf{O}Px$ is not applicable. It does not matter, morally speaking, who leaves the lifeboat, only that someone does. This is why we cannot go from the truth of $\mathbf{O}(\exists x)Px$ to the truth of $(\exists x)\mathbf{O}Px$ on my reading.

A way of looking at this is that normative propositions act as a filter over the set of possible worlds. Imagine that, from the point of view of the actual world, we started out with the complete set of possible worlds, i.e. all the worlds that are alethically acceptable, we can then conceive every normative proposition as a filter that excludes those possible worlds that would violate the normative proposition in question. It is then interesting to check which set of possible worlds a certain normative proposition would filter out when considered true at the actual world. This allows us to compare the constraints a normative formula imposes on the set of possible worlds and allows us to ask whether the set of possible worlds that would be filtered out *really* are worlds that are unacceptable deontically speaking.

In addition to the lifeboat example, Hilpinen and McNamara (2013) also give this example:

> ...the sentence 'Someone ought to rescue the cat Gussie from the shelter for abandoned pets' is ambiguous: It can be understood as having the form of the antecedent of (7.8) [$\mathbf{O}(\exists\alpha)A$] (a wide-scope ought) or the form of its consequent [$(\exists\alpha)\mathbf{O}A$], and the former interpretation does not mean that some specific person has a (personal) obligation to rescue Gussie. (Hilpinen and McNamara, 2013, p. 53)

The Gussie example is very similar to the lifeboat example. It plays on the fact that what they call the "wide-scope ought" is of a more general nature allowing an arbitrary person to fulfil the obligation at each accessible world while the "narrow-scope ought" requires only some specific persons to fulfil it. On my reading it is the wide-scope ought or the formula *de dicto* that expresses an absolute quantified norm while the narrow-scope ought or the formula *de re* expresses a person-specific obligation. This seems to be in line with how Hilpinen and McNamara interpret the situation. In this particular case, it is true that there is an absolute quantified norm in place demanding that Gussie be rescued, but it is not true that there is some specific person who has the obligation to do it. In other words, absolute existentially quantified norms are not able to induce person-specific norms.

Hilpinen and McNamara (2013) also give us an example that illustrates the conceptually problematic nature of the converse Buridan formula:

> Everyone is permitted to have a dinner in Casa Paco, a public restaurant, but no situation in which everyone is having dinner in Casa Paco is permitted (normatively acceptable), because the legal seating capacity of the restaurant is 40 customers." (Hilpinen and McNamara, 2013, p. 53)

$(\forall x)\mathbf{P}Px$ where "P" stands for "dining at Casa Paco" expresses the following: given the current state of affairs it is true that for each one of the people in the domain of the actual world there is at least one world that is not deontically ruled out at which they dine at Casa Paco. Surely this is a true description of the situation. Everyone, may, in principle, go dine at Casa Paco. Which is to say that no one is prohibited from dining there.

$\mathbf{P}(\forall x)Px$ says that given the current state of affairs it is true that there is a world that is not deontically ruled out at which everyone dines at Casa Paco. This formula requires there to be at least one possible world at which everyone dines at Casa Paco to be among the set of acceptable worlds. However, the seating capacity of the restaurant is only 40 customers and so it will not be true that such a world should be considered deontically acceptable. It follows that $\mathbf{P}(\forall x)Px$ is not true of this situation.

A similar example can be found in *Formal Ethics* by Harry Gensler:

> 1. $(x)RSx$: it's all right for anyone to stay home.
>
> 2. $R(x)Sx$: it's all right for everyone to stay home.
>
> Again, the first might hold without the second; maybe it's all right for any specific person to stay home from work today - and yet it would be disastrous

if everyone did it (Gensler, 1996, p. 173).

Thomason and Stalnaker (1968) provide yet another example that captures this distinction in scope. The difference can be thought of as the difference between "everyone can come along with us" and "anyone can come along with us". Whether one interprets the "can" as alethic or deontic, it is clear that they do not intuitively express the same thing.

The examples just discussed show that the invalidity of the converse Ghilardi formula and the converse Buridan formula is not just a fluke but a desired consequence of the way in which the models are designed. Let us now take a look at a formula that is valid on some frame conditions and so can help us decide whether some frame conditions are desired.

4.4 The Ghilardi formula and the Buridan formula

Suppose we again let "P" stand for "dining at Casa Paco". $(\exists x)\mathbf{O}Px$ then says that there exists someone in the actual world for whom it is true that this person in each acceptable world dines at Casa Paco. There are some ways in which this could be true. For example, John has promised to Paco that he will come and dine at his restaurant which would give us $\mathbf{O}Pa$. John exists at the actual world and so we infer $(\exists x)\mathbf{O}Px$. In this case, it is John who has to go to Casa Paco and not just any arbitrary person which is an obligation he has incurred upon himself by virtue of the conditional norm that if you promise something you have to fulfil it. The correct semantic correspondence seems to be that every possible world at which John exists and dines at Casa Paco should be acceptable with respect to Johns obligation.

$\mathbf{O}(\exists x)Px$ says: given the current state of affairs it is true that worlds at which no one dines at Casa Paco are unacceptable. On my reading, this expresses an absolute quantified person-non-specific norm. Semantically it says that in order for a possible world to be recognised as acceptable someone at that world has to dine at Casa Paco. It doesn't matter who, there just has to be someone who does. This kind of absolute quantified norm does not seem to follow from the fact that someone at the actual world has incurred an obligation because of a conditional norm. A world in which no one dines at Casa Paco might be unfortunate for Paco, but, surely, such a world should not be deontically ruled out by virtue of the fact that there is someone who has promised to dine at Casa Paco? There seems to be no genuine conceptual clash between saying that John ought to go dine at Casa Paco while at the same time claiming that a world in which nobody dines at Casa Paco is also acceptable.

We know that the Ghilardi formula $(\exists x)\mathbf{O}Px \supset \mathbf{O}(\exists x)Px$ is formally valid on increasing domain frames and I have just argued that **GF** shouldn't be valid. Now we are in a position to answer why this happens on a formal level. Increasing domains ensure that the person for whom it is true that s/he dines at Casa Paco in each acceptable world is also in the domain of each acceptable world and so guarantees that at each acceptable world there actually exists someone who dines at Casa Paco. Increasing domains does not allow us to look at worlds in which John does not exist. It collapses the distinction between obligations incurred by virtue of a condition being met and absolute quantified norms because in order to validly assess the truth of an absolute quantified norm we have to be able to abstract away from the actual world and the people in there. If we don't do this the obligations that hold because of facts about the actual world and the people in there influence to a large extent what happens at the acceptable worlds. To allow for the situation in which it is true that there is someone for whom it is obligatory that s/he dines in Casa Paco at each acceptable world and that there is still an acceptable world at which no existing person dines in Casa Paco we have to look at models in which the domains of worlds decrease to allow for a world in which John does not exist. It follows that we should not impose increasing domain frames on our models.

It is at this point that we can also notice a peculiar consequence of the standard clause for the deontic operator. In order to assess whether John has to dine at Casa Paco, $\mathbf{O}Pa$, it also takes into account acceptable worlds at which John does not exist. The fact that this happens, as previously explained, is because this clause does not discriminate between existing and non-existing people. The Van Benthem clause performs better in this situation because it captures the idea that the actions of John at acceptable worlds where he does not exist should not matter deontically speaking. The standard deontic operator clause commits us to claiming that what existing people and non-existing people do at the acceptable worlds is equally deontically relevant. The Van Benthem clause commits us to the claim that only what the existing people do at the acceptable worlds is deontically relevant. In this sense the standard deontic operator clause commits us to more and so the burden of proof seems to lie with those in favour of the standard deontic operator clause. The deontic status of non-existent people remains obscure and so my argument consists mostly of the claim that we should not take them into account unless we have a compelling reason to do so.

To illustrate why the Buridan fomula ($\mathbf{P}(\forall\alpha)A \supset (\forall\alpha)\mathbf{P}A$) is problematic we need a situation in which it is permissible that everyone A while at the same time there being someone such that s/he ought to not A. How can this be? Suppose that Mary has just graduated from high-school and wants to go to the prom. However, her mother reminds her of the fact that she has bad grades and that if she had bad grades she wouldn't be allowed to go to the prom. This allows us to conclude $(\exists x)\mathbf{O}\neg Px$: there is someone who is obligated to not go to the prom. The question is now whether this clashes with the assertion that it is permissible that everyone goes to the prom ($\mathbf{P}(\forall x)Px$). Notice that this last assertion does not say that every individual under consideration, which would include Mary, is permitted to go to the prom but that a situation in which everyone goes to the prom is an acceptable situation. It seems that there is a perfectly good sense in which these formulas do not clash.

Just as before. The fact that Mary is not allowed to go to the prom is an obligation that she has incurred by virtue of a fact about the actual word: her having bad grades and the conditional norm that she wasn't going to be allowed to go to the prom if she had bad grades. $\mathbf{P}(\forall x)Px$, however, is a *de dicto* formula representing an absolute quantified norm that makes abstraction of the people in the actual world and simply states that a situation in which everyone goes to the prom is acceptable. So in order to make this abstraction possible we need to reject the increasing domain frame condition which allows us to look at worlds in which Mary does not exist and make it a possibility that everyone who has graduated does indeed go to the prom.

If one is convinced that making a difference between formulas that express absolute quantified norms and formulas that express person-specific obligations is sensible, the Ghilardi formula seems unwanted. This formula allows one to infer from the existence of someone with a person-specific norm to the existence of an absolute quantified person-non-specific norm and I have given some examples of situations in which this inference is unsound. The Buridan formula is equally undesirable because from the fact that a situation in which everyone, whoever they might be, A's it does not necessarily follow that everyone has the person-specific permission to A. Within the context of these considerations we can exclude two possible logics as being viable candidates: $\mathbf{QFD} + \mathbf{CBF}$ and $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$.

4.5 The Barcan formula and the $\mathbf{P}$*-Barcan formula*

If we impose the frame condition that the domains of accessible worlds are decreasing the Barcan formula and the $\mathbf{P}$-Barcan formula become valid. I will argue that there are reasons to reject those axioms and hence to reject the associated frame condition.

Suppose that there is a conditional norm to the effect that anyone who misbehaves ought to be punished ($(\forall x)(Mx \supset \mathbf{O}Px)$). Suppose furthermore that, as a matter of contingent fact, everyone has currently misbehaved ($(\forall x)Mx$). Then it would logically follow that everyone ought to be punished ($(\forall x)\mathbf{O}Px$). The semantic condition corresponding to this would be that all the people of the actual world are punished in all the acceptable worlds. Accepting the Barcan formula entails that $\mathbf{O}(\forall x)Px$ must also be true. This formula, expresses an absolute quantified person-non-specific norm stating that everyone, no matter who they be, ought to be punished. This does not seem to be the right depiction of the situation. It is a matter of contingent fact that everyone ought to be punished.

This example shows that "everyone", just like "someone", can also be used in at least two different ways within normative discourse. On the one hand, "everyone ought to A" is shorthand for saying that everyone under consideration has the person-specific obligation to A which might be true as a matter of contingent fact. On the other hand, it might also mean that the fact that everyone is obligated to A is a feature of the obligation itself: it does not admit of any exceptions and is true of every person whomsoever.

Everyone ought to reduce their ecological footprint might be the sort of norm that holds as a matter of contingent fact. Everyone ought to reduce their ecological footprint because, as the world is right now, everyone is polluting more than their fair share. Current affairs, however, might be different. Perhaps only some people are polluting more than their fair share in which case it will not be true that everyone ought to reduce their ecological footprint. In other words, "everyone ought to reduce their ecological footprint" is in such a case a person-specific norm applying to everyone of the domain and so should be captured by a universal formula *de re* and not by a person-non-specific universal formula *de dicto*.

Increasing domains allows for worlds at which there are people that do not exist in the actual world. It seems wrong that we would demand that they too reduce their ecological footprint because there are no facts about them in the actual world that warrants rendering a world unacceptable if they do not reduce their footprint there. On the other hand, a formula such as $\mathbf{O}(\forall x)Kx$ where the "K" stands for "being kind" might be the sort of norm that is applicable to everyone without exception. People ought to be kind not by virtue of how the actual world is but simply because they are human. Which is why $\mathbf{O}(\forall x)Kx$ is applicable to all the people at the acceptable worlds irrespective of whether they exist at the actual world. By also looking at how people that do not exist at the actual world behave in the acceptable worlds we can infer how it is that people will behave under ideal circumstances.

In Erica Calardo's PhD thesis "*Non-normal Modal Logics, Quantification, and Deontic Dilemmas. A Study in Multi-Relational Semantics*" we can see some similarities to the kind of view advanced in this chapter, although Calardo's view is stated in more general terms and not fully worked out:

> In fact, when we have formulae like $\Box\forall xF(x)$, obligations may leave the problem of reference (application) to existing individuals out of consideration, as the question of their concrete application is somehow put into brackets. In other words, we may state that something is obligatory for some individuals independently of any concern about concrete applicability. This is not absurd as we may argue that something is deontically correct, it ought to be [the] case, for conceivable individuals that, as far as we know, may not exist. In the second case - when we have formulae like $\forall x\Box F(x)$ - the focus is rather on the actual world with respect to which we want to state whether something is or is not obligatory. (Calardo, 2013, pp. 85-86)

She summarises "In general, if we adopt the actualist interpretation of quantifiers, it seems to us that de dicto and de re deontic sentences may correspond, respectively, to non-contextual (or generic) and contextual (or concrete, actual) obligations." (Calardo, 2013, p. 85). This seems somewhat in line with my account, however, due to a lack of examples it is hard to know what she really has in mind.

Let us try to set up a similar situation for the $\mathbf{P}$-Barcan formula ($\mathbf{P}(\exists\alpha)A \supset (\exists\alpha)\mathbf{P}A$). Suppose that it is in general permissible to eat candy which is to say that there is no obligation to not eat candy. We can therefore conclude that $\mathbf{P}(\exists x)Cx$ because a situation in which there is someone who eats candy is acceptable. However, there is a caveat, unhealthy people are not allowed to eat candy $((\forall x)(Ux \supset \neg\mathbf{P}Cx))$. The actual world happens to contain only unhealthy people $((\forall x)Ux)$. We are therefore allowed to infer that $(\forall x)\neg\mathbf{P}Cx$ which in turn gives us $\neg(\exists x)\mathbf{P}Cx$. So while it is not the case that any existing person is allowed to eat candy, a situation in which someone eats candy will not be rendered unacceptable by default. In other words, there is an absolute quantified norm that permits a situation in which people eat candy but, as the world is right now, there is no existing person that has the person-specific permission to eat candy.

Historically, this formula has been remarked upon in *Time and modality* in 1957 by Arthur N. Prior when he claims that "Mr. P. T. Geach has suggested that intuitively the assertion that it is permissible that there should be someone ϕ-ing $(P\Sigma x\phi x)$ is weaker than the assertion that there is someone to whom it is permissible to ϕ $(\Sigma xP\phi x)$" (Prior, 2003, p. 142). Prior furthermore clarifies that "...perhaps no existing individuals are permitted to do a certain thing, though there might in a different state of affairs have been individuals who were."(Prior, 2003, p. 144).

If one is convinced that making a difference between formulas that express absolute quantified norms and formulas that express person-specific obligations is sensible, the Barcan formula also seems unwanted. This formula allows one to infer from the fact that everyone has the person-specific obligation to A to the existence of an absolute quantified norm demanding that everyone A. Just as before, I have given some examples that show that this inference is not desirable. The $\mathbf{P}$-Barcan formula should not be valid because it does not follow that there exists a concrete individual who is permitted to A from the fact that a situation in which someone A's is acceptable. Within the context of these considerations we can exclude two possible logics as being viable candidates: $\mathbf{QFD} + \mathbf{BF}$ and $\mathbf{QFD} + \mathbf{BF} + \mathbf{CBF}$.

4.6 The converse Barcan formula and the converse $\mathbf{P}$-Barcan formula

As proven in section 3.4, if we use the standard deontic operator clause we have to impose the frame condition that the domains of accessible worlds are increasing to validate the converse Barcan formula and the converse $\mathbf{P}$-Barcan formula. I will argue that there are reasons to accept those axioms but nevertheless reject the associated frame condition. The only way in which we can do this is to reject the standard deontic operator clause and accept the Van Benthem clause which allows us to reject the increasing domains condition while still validating the converse Barcan formula and the converse $\mathbf{P}$-Barcan formula.

Let us first inquire into its intuitive meaning. As discussed before, its antecedent is a wide-scope ought suggesting that it is indicative of an absolute quantified norm. $\mathbf{O}(\forall x)Kx$, or in words, it ought to be the case that everyone is kind will again do as such an example. Given my reading, the converse Barcan formula asserts that this absolute quantified obligation logically entails that everyone in the actual world has the absolute person-specific obligation to be kind. Intuitively this makes sense. If everyone in the domain of every acceptable world is kind then surely this suggests that the people living in the actual world ought to be kind. Or in other words, if there is a norm to the effect that everyone, no matter who they be, ought to be kind then surely everyone that exists has the person-specific obligation to be kind ($(\forall x)\mathbf{O}Kx$).

If the converse Barcan formula seems intuitively justified it is strange that it is only valid if we impose the increasing domain frame condition because I have previously argued that we do not want the increasing domain frame condition because this would validate the Ghilardi formula which, I have argued, should not be valid. The reason why the converse Barcan formula is invalid when rejecting the increasing domain condition is the consequence of the way in which the deontic operator is defined. The standard deontic operator clause does not discriminate between the existence and the non-existence of a person in the acceptable worlds and therefore leads to strange results. The invalidity of the converse Barcan formula is tied to the standard formulation of the deontic operator because every model intended to falsify the converse Barcan formula will exploit its defect by using non-existent people to falsify the consequent of the formula. For example, take $\mathbf{O}(\forall x)Px \supset (\forall x)\mathbf{O}Px$. If the antecedent is true we are guaranteed that at every acceptable world every existing person there is in the extension of P. In order for the consequent to be false we have to find a person in the actual world that is not in the extension of P at one of the acceptable worlds. However, every existing person at the acceptable worlds necessarily will be in the extension of P. The only option left to falsify the consequent $(\forall x)\mathbf{O}Px$ is to search for the acceptable world at which someone from the actual world does not exist and is not in the extension of P. The invalidity of the converse Barcan formula thus hinges completely on what non-existent people do in the acceptable worlds. Because the converse Barcan formula seems intuitively justified and the Ghilardi formula unjustified this is another argument in favour of the Van Benthem clause because adopting it is the only way in which to validate the converse Barcan formula while invalidating the Ghilardi formula.

The converse $\mathbf{P}$-Barcan formula allows us to infer $\mathbf{P}(\exists\alpha)A$ from the formula $(\exists\alpha)\mathbf{P}A$. If we reuse the candy example it says that if there is someone for whom it is permissible to eat candy then it must be true that a situation in which someone eats candy is acceptable. This inference, just like the converse Barcan formula itself, seems intuitively acceptable. The fact that the converse $\mathbf{P}$-Barcan formula is relatively unassuming is because its consequent $\mathbf{P}(\exists\alpha)A$ is the normatively least demanding formula: its truth requires only one acceptable world in which only one individual has to A. It follows that if we know that there is such an individual who is permitted to A then we know that there will be at least one situation in which eating candy is acceptable: the situation in which that person eats candy.

Given my reading, the converse Barcan formula seems like a valid inference and so should be considered as an axiom. This formula allows one to infer from the existence of an absolute quantified norm demanding that everyone A that everyone has the person-specific obligation to A. The converse $\mathbf{P}$-Barcan formula is also desirable because if there is someone who is permitted to A then there will exist a situation in which it is acceptable that someone A's. Within the context of these considerations we can exclude two possible logics as being viable candidates: $\mathbf{QFD}$, $\mathbf{QFD} + \mathbf{BF}$.

4.7 Summary

In this chapter, I have tried to show that a first-order deontic logic is able to capture the difference between people having person-specific obligations and the representation of absolute quantified norms. If we want a sensible logic that is able to deal with these differences, we have to give up some axiom candidates.

In section 4.3 I have shown that the invalidity of the converse Ghilardi and the converse Buridan formula is not just a fluke of the models. The existence of an absolute quantified norm demanding someone to A does not allow us to infer that there is at least one specific person that has to A. It is equally not the case that if everyone has the person-specific permission to A, a situation in which everyone A's is permissible.

In section 4.4 we looked at the Ghilardi formula and the Buridan formula. The Ghilardi formula was found undesirable on my reading because we can not infer from the fact that someone has a person-specific obligation to A that there is also an absolute quantified norm requiring someone to A. The Buridan formula is equally undesirable because from the fact that a situation in which everyone, whoever they might be, A's, it does not necessarily follow that everyone has the person-specific permission to A. The adoption of the increasing domain condition would validate the Ghilardi and Buridan formula and hence needs to be rejected.

In section 4.5 we looked at the Barcan and $\mathbf{P}$-Barcan formula. I have argued that both of these formulas should not be valid given my reading. The Barcan formula should not be valid because it allows us to infer an absolute quantified norm that everyone has to A from the fact that everyone under consideration has the person-specific obligation to A. However, the antecedent, that everyone under consideration has to A, might be true because of some contingent state of affairs and so does not guarantee the existence of an absolute quantified norm demanding that everyone A's. The $\mathbf{P}$-Barcan formula should not be valid because it does not follow that there exists a concrete individual who is permitted to A from the fact that a situation in which someone A's is acceptable. The Barcan formula and the $\mathbf{P}$-Barcan formula become valid if we adopt the decreasing domain condition. Because this is undesirable on my reading we have to reject the associated frame condition.

In section 4.6 we looked at the converse Barcan and converse $\mathbf{P}$-Barcan formula. The converse Barcan formula allows us to infer that everyone under consideration is obligated to A if it is true that there is an absolute quantified norm to the effect that everyone has to A. This is unproblematic on my reading because if some norm applies to every arbitrary person then surely it also applies to the people at the actual world. The converse $\mathbf{P}$-Barcan formula is also desirable because if there is someone who is permitted to A then there will exist a situation in which it is acceptable that someone A's. The validity of the converse Barcan and converse $\mathbf{P}$-Barcan formula is tied to the frame condition that domains increase when using the standard deontic operator clause. This clashes with the undesirability of the Ghilardi formula because we need to reject the increasing domain condition to invalidate it. However, I have shown that the invalidity of the converse Barcan and converse $\mathbf{P}$-Barcan formula hinges on what non-existent people do at the acceptable worlds. Luckily, this can be remedied by adopting the Van Benthem clause which captures the fact that only what existing people do at the acceptable worlds is deontically relevant to the people at the actual world.

All of the above considerations lead me to the view that given my reading we do not want to impose any conditions on the frame of the models and so we will end up with varying domain models. Moreover we want to adopt the Van Benthem clause for two reasons. Firstly, because the standard deontic operator clause is intuitively strange due to the fact that we are working with a free logic. It claims more than the Van Benthem clause and we have no compelling reason to accept its claim that non-existent people matter deontically speaking. Secondly, because it leads to the validity of the converse Barcan formula and converse $\mathbf{P}$-Barcan formula on varying domain models. Unfortunately, none of the logics that I have introduced in section 3.3.3 is sound with respect to varying domain models and the Van Benthem clause.

5. CONCLUSION AND FUTURE RESEARCH

In this final chapter, I will first summarise what I have done so far and present my conclusions. Thereafter I will highlight the merits of this thesis, some of its limitations and shortcomings and present some options for future research.

In chapter 3 I have explicated some of the formal details of quantified deontic logics. We learned that in order to preserve generality we are forced to adopt the principles of free logics. This, in turn, prompts us to consider a revision of the clause for the deontic operator because it takes into account what non-existent people do at the acceptable worlds which is intuitively hard to justify. The revision of this clause was dubbed the Van Benthem clause and it makes sure that what non-existent people do at the acceptable words has no influence on the truth of a deontic formula at the actual world. I have introduced the logic **QFD** and the axiom candidates which we can use to extend this logic: the Barcan formula, the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula. On the semantical side, I presented varying domain models and three types of frame conditions that we can impose: increasing domains, decreasing domains and constant domains. We learned that these frame conditions can be used to validate or invalidate some of the axiom candidates that regulate the interaction between formulas containing quantifiers and deontic operators. I will not repeat all of these relations here because it would be tedious and they have already been summarised in section 3.4.5. The major takeaway from this chapter is perhaps that if we change the standard deontic operator clause to a Van Benthem clause, we are suddenly able to validate the converse Barcan formula while at the same time invalidating the Ghilardi formula. This is impossible if we use the standard deontic operator clause. However, there is a catch, adopting the Van Benthem clause leads to a non-normal modal logic and it is as of yet unclear what this entails precisely.

In chapter 4 I have introduced a distinction between person-specific norms and person-non-specific norms. I have argued that we can read an existentially quantified formula *de re* as saying that there is someone i.e. at least one person with a specific obligation/permission while the universally quantified formula says that everyone has the person-specific obligation/permission in question. In contrast to these person-specific obligations and permissions, there also seem to be person-non-specific obligations and permissions. These are what I have called absolute quantified norms and they can be captured by using formulas *de dicto* either existentially quantified or universally quantified depending on the intention. By interpreting these formulas expressing interactions between deontic operators and quantifiers in this way, we are forced to consider which axiom candidates to adopt that regulate this interaction. I concluded that we have to give up the Barcan formula, the Ghilardi formula and the converse Ghilardi formula and their syntactic equivalents. The only acceptable axiom candidate is the converse Barcan formula. In order to make it a semantic possibility that we can validate the converse Barcan formula while invalidating the Ghilardi formula we have to adopt varying domain models and the Van Benthem clause.

Unfortunately, there remain at least two open problems with respect to varying domain models and a Van Benthem clause. The first one is what its precise semantic properties are. I have demonstrated that the **KD** axiom scheme is not valid but it is not yet clear which other principles are preserved and which are not. If we discover that we have to give up some desirable principles without having a proper substitute then perhaps we are forced to conclude that the Van Benthem clause is undesirable after all. The second open problem is how to axiomatise this semantics so that we have a logic that is sound and complete with respect to it.

The merits of this thesis lie mostly in the fact that I have brought attention to the role of quantifiers in deontic logics. This was done by explicating a variety of possible first-order deontic logics and proving some of their formal properties. Although most of these properties were already known I have taken the time to fully present their proofs. As far as I know, this is the first time that attention has been paid to the way in which the deontic operator clause is semantically defined within first-order deontic logics. This thesis also contains the first defence of revising this deontic operator clause in favour of a clause that is able to deal with the fact that some first-order deontic logics end up as free logics. This is also the first study that has explicitly and systematically advanced an interpretation of the Barcan formula, the converse Barcan formula, the Ghilardi formula and the converse Ghilardi formula when given a particular semantic interpretation.

However, this thesis is not without its shortcomings. I have advanced an account that interprets some formulas as being about person-specific norms while others capture some type of person-non-specific norms. However, all of my examples were fictional. My account would have perhaps been more persuasive if it would be backed up by some empirical work that established that there are indeed person-specific norms and person-non-specific norms and that some expressions are ambiguous between these two interpretations. Another shortcoming is that I have argued that we should adopt varying domain models and use a Van Benthem clause while it is as of yet not clear what this entails precisely. We know that we will end up with a non-normal modal logic but I have not provided arguments as to whether this is a good thing. However, before such an investigation can be done it is important to first find out what formulas it validates and how to axiomatise it. The two shortcomings that I have outlined here can thus be considered as possible future research. This leads me to consider some other possibilities with respect to future research.

There are, for example, some questions left with respect to substitution and identity within "ought"-contexts. Lou Goble has argued in Goble (1996) and Goble

(1994) that deontic modalities are *extensional*. What he means by this is that co-referring terms can be substituted *salva veritate* within the scope of the deontic operator. There is some reason to think that this is unproblematic within deontic contexts. For example, suppose that Superman ought to rescue a child and we learn thereafter that Superman is in fact Clark Kent then it seems non-problematic to infer that Clark Kent ought to rescue a child. However, this threatens triviality if we take into account definite descriptions. Goble's argument goes as follows. Take a true statement $\mathbf{O}Fa$ now if $a = \iota x(x = a \wedge p)$ where p can be any truth it then follows, if we can substitute *salva veritate*, that $\mathbf{O}F\iota x(x = a \wedge p)$ must be true and then it follows that $\mathbf{O}p$ (see Goble (1996) p. 324 for the full explanation and his solution). In general, the nature of definite descriptions, identity and substitutivity within first-order deontic logic remains unclear. It would be an interesting route to further explore this problem and perhaps investigate whether semantically defining the deontic operator by a Van Benthem clause brings anything new to the table.

Whereas Goble argues that first-order deontic logic is extensional, Federico Faroldi has argued that it is *hyperintensional*. An operator $\mathbf{O}$ is hyperintensional if, for example, $\mathbf{O}A$ and $\mathbf{O}B$ can have different truth values even though A and B are necessarily equivalent (Berto, 2017). Faroldi gave the following example at the workshop "Logic in Bochum III". We can have "The pope must shake hands with Shakira" and "Shakira must shake hands with the pope". These two statements do not seem to express the same and so it is conceivable that the one is wrong while the other is true. However, "Shakira shaking hands with the pope" or "the pope shaking hands with Shakira" are two necessarily equivalent statements and so within standard Kripke possible world semantics this implies that "The pope must shake hands with Shakira" and "Shakira must shake hands with the pope" will also be equivalent statements. In order to make these distinctions, however, we need to leave standard Kripke semantics behind and adopt a different kind of semantics. Faroldi does this by using a type of truthmaker semantics (see Anglberger et al. (2016) to see how this type of semantics works at the propositional level). It would be interesting to further explore whether first-order deontic logic is in fact hyperintensional, what this entails precisely and what kind of semantics we need to capture it.

Although the last word has definitely not been said about first-order deontic logic within the context of standard relational Kripke semantics it might prove insightful to contrast Kripke semantics with other types of semantics. As I have hinted at before, counterpart semantics is one such plausible candidate. For starters, within counterpart semantics the converse Barcan formula and Ghilardi formula correspond to different semantic conditions which is not the case within Kripke semantics if we use a standard deontic operator clause (Corsi, 2002*a*). This is interesting from the point of view of the interpretation advanced in this thesis because I argued that we want to validate the converse Barcan formula while invalidating the Ghilardi formula. Because counterpart semantics is a generalisation of Kripke semantics this gives us more formal flexibility. Giovanna Corsi has argued in Corsi (2003) that "A major step forward to the clarification of the meaning of BF was achieved by counterpart semantics, C-semantics. [...] As we shall see, in counterpart semantics the meaning of BF is well captured, whereas the meaning of CBF still remains opaque. We will introduce a generalization of counterpart semantics that we call Lewis semantics, L-semantics, to address this problem." (Corsi, 2003, p. 103). Of course, the question remains whether the formal flexibility that these types of semantics offer is a good thing and captures the meaning of quantified deontic sentences better. It would be interesting to investigate what these types of semantics can offer and whether they can show us something about the limitations of Kripke semantics from the deontic point of view.

The possibilities that I have briefly sketched above are just some of the options before us. Suffice it to say that this thesis is only a stepping stone towards a more extensive analysis of the role of quantifiers in deontic logics. As Gabbay et al. (2009) put it "Up to now the focus was mainly propositional. Now the era of the quantifier has begun!" (Gabbay et al., 2009, p. xii). Let us hope that this sentiment extends to deontic logics as well.

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