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Knowability as real knowledge potential

A dynamic approach to knowability

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1 Introduction

Adam is living his best life experiencing reality and interacting with the world around him. He is convinced he knows every little corner of his universe. Unbeknown to him however the world he lives in is but the shadow of a deeper reality. Unfortunately for Adam, he is trapped in Plato's cave and has little knowledge of the real world. Yet in Plato's allegory of the cave, Adam can be hopeful in his ignorance, that even if there are things about his world he does not yet understand, one day he may escape the confounds of the cave and see reality for what it truly is. This silver lining perfectly encapsulates the verification postulate that all truths are knowable, that at one point in time, an agent may know them.

Verificationism is a philosophical tenet that tethers the meaning of a statement to the ability to verify its truth. According to the verification principle, all statements must be verifiable either a priori for logical truths or a posteriori for empirical truths. There is therefore no room for hidden truths, sentences that are true despite being unverifiable. Verificationists, however, must allow for unknown truths, those being verifiable truths that have yet to be verified. The clash between hidden and unknown truths, between the verification principle that all truths are knowable and the existence of hitherto unknown truths, is the focus of the Fitch paradox of knowability.

The opponents of verification argue that there may be truths that cannot be known and that the cave may prove to be inescapable. One of their greatest arguments is the Fitch paradox of knowability (Fitch, 1963). The paradox is a simple logical deduction that seemingly dismantles the verification postulate by showing it to be contradictory to the assumption that there are unknown truths. The argument first translates the verification thesis into formal modal logic. Using standard semantics the thesis "all truths can at one point be known by some agent" can be interpreted as:

$$\phi \rightarrow \Diamond K\phi \qquad \text{(possible knowledge)}$$

where ϕ is a variable ranging over formulas.

The above formalization seems to elegantly capture the intuition that at some possible world Adam escapes the cave and discovers any unknown truths. However, in combination with the basic axioms of epistemic relations, it entails a contradiction:

- $K(\phi \wedge \psi) \rightarrow K\phi \wedge K\psi$ (distribution of knowledge over conjunction)
- $K\phi \rightarrow \phi$ (factivity of knowledge)
- $p \wedge \neg Kp$ (unknown truth)

Proof.

$p \wedge \neg Kp$	(assumption of unknown truth)
$\Diamond K(p \wedge \neg Kp)$	(possible knowledge)
$\Diamond(Kp \wedge K\neg Kp)$	(distribution of knowledge over conjunction)
$\Diamond(Kp \wedge \neg Kp)$	(factivity of knowledge)
$\Rightarrow \neg(p \wedge \neg Kp)$	(no unknown truths)

□

The assumption that no truths are hidden leads to the conclusion that no truths are unknown.

While Adam can learn most truths by escaping the cave of shadows, he may never know that he is in the cave. Knowing that truth requires escaping, at which point it is no longer true. Adam can never know that the reality he is observing is a facsimile. Therefore the only way for Adam to satisfy the verification postulate, and learn that he is a cave of ignorance, is to be in the field of knowledge to begin with. Under the postulate of possible knowledge, there can be no unknown truths, only omniscience.

Naturally, verificationists are not keen on abandoning their project. Many attempts to overcome the paradox have been made, striking at different parts of the proof. In this paper I will focus mostly on reinterpretations that reject the translation of the core intuition "all truths can at one point be known by some agent", into possible knowledge. I will tackle the demands that a correct interpretation must satisfy before proposing a dynamic solution, knowability as real knowledge potential.

2 Attempted reinterpretations and their problems

2.1 The hope of reinterpretation

The possible knowledge formalization leads to the untenable conclusion that all truths are known. Despite the severity of such a conclusion, verificationism can hope to persevere under different interpretations. Since the various suggested reinterpretations get increasingly complex, I will first give some examples of trivial solutions that, while having other problems, successfully bypass the Fitch paradox.

Possible omniscience

While not a mainstream solution, possible omniscience could technically be considered a reinterpretation and is the first indication that a different formalization of knowability can overcome the Fitch paradox. If we entertain possible omniscience as a proposal, then knowability merely amounts to the possibility of a world where every proposition true in it, is known to be true.

$$\Diamond\forall\phi(\phi \rightarrow K\phi) \qquad \text{(possible omniscience)}$$

Possible omniscience does capture, albeit in a trite sense, the intuition that Adam may escape into the valley of knowledge. Unlike possible knowledge, the postulate of possible omniscience does not contradict the assumption of unknown truths.

This simple band-aid solution might overcome the problem of actual omniscience but at too heavy a cost. Whatever the agent that may possibly reach the coveted plane of knowledge, he most surely cannot be human. As Heylen (2020b) argues, empirical knowledge of the location of every atom in the universe is a feat beyond human comprehension. Similarly, the a priori knowledge of the solution of all undecidable problems, that while having a definitive truth value cannot be computed, is a ridiculous demand for a finite agent. While logically consistent, possible omniscience cannot adequately support the intuition of verificationism which concerns itself with human understanding. Possible omniscience will be one of the main pitfalls of aspiring reinterpretations, an undesired consequence that in the same vein as actual omniscience, must be circumvented. Yet, at the very least, it demonstrates that unknown truths can be compatible with a different notion of knowability.

Knowledge of ontic truths

Moving away from omniscience, possible or actual, a simple way to bypass the Fitch paradox is to limit the domain of knowable propositions (Brogaard and Salerno, 2019). Many restriction strategies have been suggested, one of which is to exclude epistemic propositions and only demand the possible knowledge of purely ontic ones. An ontic proposition is a formula that describes the state of affairs and is devoid of epistemic operators. On the contrary epistemic propositions employ knowledge operators such as K to express an agent's beliefs regarding the world.

Under this restriction, the new verification formalization becomes:

$$p \rightarrow \Diamond Kp \qquad \text{(knowability of ontic truths)}$$

where p is an ontic proposition.

This move is motivated by the observation that a change from ignorance to knowledge necessarily requires a change of valuation for epistemic propositions. The Fitch sentence demands a fixed valuation and thus can never be known to be true. The knowability of ontic truths signals out Fitch sentences and other linguistic artifacts for what they are, word-play, and crudely pushes them out of sight and out of mind. In its attempt to swiftly sever the Gordian knot, however, this interpretation strikes blindly and indiscriminately against all epistemic propositions. Valuable propositions like knowledge of introspection[1] are sacrificed for the sins of the Fitch sentence. Yet intuitively, it should be knowable to Adam that he does not know he is in a cave. Limiting knowability to ontic truths illustrates another demand for a proper formalization, that it should apply to all true propositions.

Knowledge whether

An interesting remark is that the Fitch sentence is only unknowable while true. When Adam gains knowledge of a previously unknown truth p, its respective Fitch sentence $p \wedge \neg Kp$ changes evaluation. With a simple introspection, Adam can indeed know that the Fitch sentence is false. He can thus assert whether it is true or not. The following interpretation tries to harness this observation.

$$\Diamond(K\phi \vee K\neg\phi) \qquad \text{(knowledge whether)}$$

As evident by its name, this formalization of the verification thesis claims that for every truth it is possible to know whether it is true. While it swiftly disposes of the paradox, this solution too proves to be a double-edged sword. In the example of the cave, knowledge-whether amounts to the underwhelming development of Adam leaving the cave of ignorance and learning that he is not in the cave of ignorance. This formalization trivializes the acquisition of truths by considering worlds where they are falsehoods.

Imagine a proposition that is easily known when true but difficult to attain when false. Perhaps Adam questions whether he is mortal. If he is, then one day he will draw his last breath and know of his mortality. If he is not, then he will live every day wondering if it is his last. Is Adam's mortality knowable to him? Knowledge-whether immediately asserts positively as it is always possible, broadly speaking, that he should be mortal and thus

[1] Introspection refers to knowledge about knowledge, formally $KK\phi$ or $K\neg K\phi$

know to be mortal.

The problem this time stems from the loss of factivity. Factivity is the property of knowledge to only apply to truths. Only true propositions can be known and similarly, only true propositions should be knowable. In the knowledge-whether interpretation, all propositions are knowable regardless of their truth value. Adherence to factivity will be another benchmark for aspiring reinterpretations.

Knowledge of other agents

The most promising direction against the paradox of knowability stems from the observation that the verification intuition does not specify which agent should arrive at knowledge. While the same agent cannot know the Fitch sentence, a second agent can know that the first satisfies the Fitch sentence.

$$\phi \rightarrow \Diamond K' \phi \qquad \text{(knowledge of other agents)}$$

for some agent K'.

In the example of the cave, Adam can never know that he is trapped in the cave, for he has to escape to discover it, at which point the sentence will no longer be true. It is possible, however, for Eve to escape to the fields of knowledge, at which point she will come to know that Adam is trapped in the cave of shadows without knowing it.

Most attempts at reinterpretation try to leverage this observation to bypass Fitch sentences. There are however multiple ways to utilize other agents to define knowability. Before I venture to apply it in my own reinterpretation it is necessary to elaborate on a few of the major suggestions that inspired my solution.

2.2 Non-actual knowledge of actuality

Adam can never know the proposition ϕ: "p and I don't know that p". But Eve may one day discover that ψ: "p and Adam does not know that p". The two propositions have the same reference to the state of affairs, so when Eve learns ψ, the knowability demand of ϕ is satisfied, even if Eve cannot express her knowledge by uttering ϕ. Propositions like ϕ are tensed and can change their evaluation depending on the context of utterance, while ψ is tenseless and its truth is invariant to changes in the evaluator.

Edgington (1985) attempts to bind knowability to tenseless propositions and thus demands that the reference world of all knowledge evaluations remains the same. Instead

of truth in a tensed "this world", she requires a fixed actual world. However, that would mean that the knowledge status of all agents in that world would also remain immutable. To express the acquisition of knowledge, an agent in a different, non-actual, world could evaluate propositions of the actual world. Such is the solution suggested by Edgington (1985) who reinterprets knowability as non-actual knowledge of actuality:

$$A\phi \rightarrow \Diamond KA\phi \qquad \text{(non actual knowledge of actuality)}$$

where A is the actuality operator.

The hope is that since Kp and AKp have different evaluations for non-actual worlds, the Fitch paradox is blocked. Indeed $KAp \wedge A\neg Kp$ is consistent for non-actual worlds.

The problem with Edgington (1985)'s solution is the difficulty of defining knowledge of other worlds. According to the semantics of the A operator, Ap holds for all worlds or none, thus leading back to omniscience $Ap \rightarrow KAp$ (Heylen, 2020b). To circumvent this issue, non-actual knowledge of actuality demands two-dimensional semantics where every word has its own reference to a perceived actual world. Once again, however, one can ask how the agent in every world comes to possess *de re* knowledge of the actual situation, in the form of a unique pointer to the actual world.

A first suggestion is through *de dicto* knowledge, using a description of the world in the form of a set of propositions true in it. Edgington (1985) gives the example of a meteor crossing the sky and an observer looking up to the visual evidence that indeed a meteor crossed the horizon. The observer now knows that in a different world where he never looked to the stars, the meteor passed without anyone ever knowing it was there. He thus knows a truth about a non-actual world. The alternate world can be described without indexical references as one where the observer never looked up, but is otherwise identical in all other propositions.

Descriptions however are universal, meaning that all worlds with the same description would arrive at the same subset of worlds as candidates for actuality, and thus knowledge of what is true in the actual world (Williamson, 2000). The observer already knows that in a world where a meteor passed while he was not looking, he would not know that a meteor passed. Adam already knows that in all worlds where he is trapped in the cave of ignorance, he does not know he is in a cave of ignorance.

A further dimension would have to be introduced, allowing each world to have its own description of actuality. The resulting endless circle of demeaning *de re* knowledge through *de dicto* description undermines the ability of Edgington's proposal to capture the verification intuition (Edgington, 1985).

2.3 Knowledge of counterfactuals

Immediate attempts to salvage Edgington (1985)'s proposal using world descriptions have been shown to lead to omniscience. Schlöder (2019) tries to improve upon descriptions by using the concept of counterfactuals. Drawing from Edgington's key intuition and the example of the meteor, he understands knowability as knowledge of slightly different worlds. The key addition is the counterfactual operator $\Box\!\!\rightarrow$. The proposition $\phi \Box\!\!\rightarrow \psi$ is true in w if and only if $\phi \rightarrow \psi$ is true in all ϕ worlds w' that are closest to w.

In his definition, Schlöder (2019) splits possible worlds into two categories based on a possible line of inquiry, those who have successfully completed it and those who have not. In the example of the meteor, there are the worlds where the observer looks to the stars and the worlds where he does not. A proposition p then is knowable if there is such a line of inquiry that even though not followed, in the closest worlds where it is followed it would be known that "in the closest worlds where it is not followed, p is true". In this world, Adam never leaves the cave and does not know of his ignorance, but in the closest world where he does, he would know that had he never left he would be oblivious of his ignorance.

This statement is formalized using nested counterfactuals. The first depicts one of the closest worlds, w_2, where the line of inquiry was successful. The second refers to the closest neighbors of w_2 where the inquiry was unsuccessful. Formally:

$$A\phi \rightarrow \exists i \in Inq(sp(i) \Box\!\!\rightarrow K(\neg sp(i) \Box\!\!\rightarrow \phi)) \qquad \text{(counterfactual knowledge)}$$

where Inq is the set of all lines of inquiry and $sp(i)$ expresses that line of inquiry i has been successfully pursued.

Schlöder (2019)'s suggestion lives and dies by the counterfactual operator. Using counterfactuals, the knowability of ϕ consists of the claim that all leaps from a starting $\neg sp(i)$ world to adjacent $sp(i)$ worlds, arrive at worlds where it is known that the leap back to $\neg sp(i)$ finds only ϕ worlds. Hopefully, through this series of world changes, we can get back to the original world, lest the interpretation be non-factive. If, while jumping from world to world, we get lost in the space of possibilities then the truth of ϕ in w is not assured, and factivity is not guaranteed.

It all depends on the interpretation of "close worlds" in the notion of counterfactuals. If it is either too broad or too narrow then the definition of knowability fails. As Heylen

(2020a) points out, to conduct a safe journey among possible worlds all the way back to the original and preserve factivity, the relation of close worlds must be broad enough to include symmetries. At the same time, if it includes too many possible worlds, then the last counterfactual $\neg sp(i) \boxdot\!\!\rightarrow \phi$ is unlikely to hold. Imagine Adam escaping to the fields of knowledge where he knows that had he not left the cave he would live in ignorance. Yet there could be a close world where Eve escaped instead and then returned to confer her newfound knowledge to Adam. In such a world it would be the case that $\neg sp(i) \wedge \neg\phi$ as Adam would not complete the line of inquiry "leave the cave" yet the proposition "Adam is oblivious" would no longer be true. Such possible worlds must somehow be excluded from the closest ones.

In the end, answering the question of which worlds are regarded as close regresses to the original problem in Edgington (1985)'s proposal. The vagueness of counterfactuals makes this solution tempting as there is always a notion of closeness that neatly captures the intuition of knowability.

2.4 Potential knowledge

The solutions I have covered so far share one element in common. They try to to reinterpret knowability as knowledge belonging to the agent in a different possible world, about the original world. In the new world, the agent sees different epistemic possibilities thus avoiding the Fitch paradox. To use a different epistemic relation, however, one does not need to venture so far. Fuhrmann (2014) offers a different approach to avoid the problem of returning to the reference world. Instead of acquiring knowledge by moving to different possible worlds, he suggests a move to a different point in time. The resulting proposal is knowability as potential knowledge.

While the Fitch sentence $\phi \wedge \neg K\phi$ cannot be known by the same agent K, a different agent K_2 can know that $K_2(\phi \wedge \neg K\phi)$. Inspired by belief revision, Fuhrmann (2014) posits that with time an agent can change his epistemic state. An epistemic agent at moment t can be modeled by knowledge state K_t. The temporal indexing enables the potential knowledge of Fitch sentences of the form $\phi \wedge \neg K_{t_1}\phi$. By moving to t_2 with $t_1 < t_2$, where the belief state of the ag ent has changed to K_{t_2} such that $K_{t_2}(\phi \wedge \neg K_{t_1}\phi)$, the truth of his prior ignorance can be known.

My proposal of real knowledge potential tries to improve upon Fuhrmann's potential knowledge. For that reason, I will briefly present its main points. To formally define po-

tential knowledge, Fuhrmann (2014) introduces a new modal operator $\langle K \rangle$. The semantic definitions are as follows:

The language of potential knowledge *LPK* is an extension of modal logic. It uses the following symbols:

- a set P of atomic propositions p
- the logical connectives $\neg, \wedge, \rightarrow, \top$
- K_i for epistemic agents
- $\langle K_i \rangle$ for potential knowledge
- $\Box, \Diamond$ for the modal operators.

The set of well-formed formulas *FORM* is defined inductively as:
$FORM := p \in P \mid \top \mid \neg\phi \mid \phi \wedge \psi \mid \phi \rightarrow \psi \mid \Box\phi \mid \Diamond\phi \mid K_i\phi \mid \langle K_i \rangle\phi$

The semantic structure, or frame, S is a quadruple containing the following:

- a set W of possible worlds w
- a binary relation N in $W \times W$, where W is the set of possible worlds, modeling metaphysical possibility
- a binary relation R_i in $W \times W$, for the epistemic state of every agent i
- a binary relation $<<$ in $W^2 \times W^2$ modeling the progression of epistemic states.

The relation $<<$ describes how agents gain –or lose– knowledge. With new evidence, their epistemic state transforms into one that can distinguish between hitherto indistinguishable worlds. Thus, a new relationship is required to model the newfound state of knowledge.

A valuation V is a function: $W \times P \rightarrow \{0,1\}$.

A model M is a pair (S, V) of a structure S and a valuation V.

Truth of a proposition ϕ in a world w in a model M, denoted by $w \models_M \phi$ or $w \models \phi$, is defined by expanding V as follows:

- if $\phi \in P$ then $w \models \phi$ iff $V(w, \phi) = 1$
- $\forall w \in W : w \models \top$
- $w \models \neg\phi$ iff $w \not\models \phi$
- $w \models \phi \rightarrow \psi$ iff, if $w \models \phi$ then $w \models \psi$ (similarly for other connectives)
- $w \models \Diamond\phi$ iff $w' \models \phi$ for some $w' \in W$ such that $N(w, w')$
- $w \models \Box\phi$ iff $w' \models \phi$ for every $w' \in W$ such that $N(w, w')$
- $w \models K_i\phi$ iff $w' \models \phi$ for every $w' \in W$ such that $R_i(w, w')$
- $w \models \langle K_i \rangle \phi$ iff there is an $R \in W \times W$ with $R_i << R$ and $w' \models \phi$ for every w' such that $R(w, w')$.

The verification thesis can be reformulated as the postulate that any given proposition ϕ true to that world can be potentially known by some agent K_i. Such an agent can undergo an evolution of his epistemic state $R_i << R'$ that results in knowledge of ϕ. Formally:

$$\phi \rightarrow \langle K \rangle \phi \qquad \text{(potential knowledge)}$$

for some K.

Potential knowledge of the Fitch sentence is consistent for the same reason that $K_1(\phi \wedge \neg K_2\phi)$ is, and its consistency can be trivially demonstrated by a model. Under some further frame restrictions, potential knowledge can be shown to be valid in all valuations. An example of such a frame condition is: $\forall R : R << I$ where I is the identity relation on $W \times W$. Since I as an epistemic relation describes omniscience, potential knowledge is trivially valid. Evolving into the identity relation, however, is not necessary for the validity of potential knowledge. A more lenient restriction can provide the same result.[2]

Furthermore, potential knowledge can be factive. A simple restriction to ensure factivity is: $\forall R$ if $(R_i << R)$ then R is reflexive. If all potential knowledge states are factive then the evolution of knowledge can never know falsehoods.

[2] Heylen (2020b) demonstrates that weaker frame conditions also lead to potential omniscience

Lastly, potential knowledge can be achieved for all propositions ϕ, including the Fitch sentence. It seems that reinterpreting the verification thesis as potential knowledge solves most of the issues that plague other attempts. Unlike non-actual knowledge of actuality, potential knowledge remains fixed in the actual world and thus does not need to justify knowledge of other worlds. Yet this very fixation to the actual world throughout the process of epistemic evolution proves to be the main vulnerability of potential knowledge.

2.5 Knowability as change

Returning to the example of Adam, as long as he is trapped in the cave of shadows, he lives in ignorance. For knowledge to replace oblivion, a change is needed. Let us interpret Adam's position as $M, w \models \neg Kp$. The evaluation of $M, w \models \neg Kp$ depends on a number of factors: the proposition ϕ, the world of reference w, the epistemic relation R of K and in general the entire model M consisting of frame S and valuation V.

To describe his original state of ignorance, w is the world where Adam is in the cave, and p is the atomic proposition "Adam is in the cave of ignorance". Moreover, K describes Adam's knowledge and is interpreted by an epistemic state R that cannot dismiss all $\neg p$ worlds. Valuation V assigns truth to p at w, and the model M includes other possible worlds with different valuations. For Adam to transcend his ignorance one of the above must change, but what exactly? At first glance, any of them is a promising candidate.

A change in the model is the most radical solution. In a different frame with a different set of possible worlds W', perhaps $W' = \{w\}$, Adam's ignorance trivially dissolves. Likewise, a change of valuation could filter out $\neg p$ worlds, illuminating the truth of p. However, the point of having a model is to institute a framework within which the acquisition of knowledge can be understood. Changing the foundations underlining the propositions we wish to learn makes little sense. In a different model, Adam is a lemur in Madagascar, knows very little about anything beyond his island, but correctly believes he is not in a cave. That is hardly what verificationism has in mind in demanding that Adam may come to know ϕ.

For similar reasons, a change of p is equally unsatisfactory. Imagine a general sending a scout out for reconnaissance. Upon his return, the general asks him the position of the enemy troops, only for the soldier to reply "I do not know that, but the cherry trees are blossoming this time of year". Would that be a fulfilling acquisition of knowledge?

Changing p trivially solves the paradox of knowability but is an unfulfilling solution.

That leaves only two possible alternatives. Changing the evaluating world w or the relationship R.

2.6 The need for actual change

Of the interpretations I have discussed so far, Edgington (1985)'s proposal of non-actual knowledge of actuality employs a change in worlds. Fuhrmann (2014)'s more successful potential knowledge keeps the actual world the same while using changes in epistemic states. Despite the obstacles of defining knowability through a change in worlds, such as knowledge of other worlds, I will now argue that such a move is necessary, and while potential knowledge is an elegant solution it is ultimately untenable.

The core issue with unilateral epistemic change is that it requires a transition to a new state of knowledge, without a change in ontic truths. Somehow the agents can be isolated from the world and adapt their belief states irrespective of the state of the actual world. From a dualistic perspective that separates the mind from the material world, such a position could be maintained. For physicalism, however, any mental state, including belief or knowledge states, is grounded on the physical facts that describe the actual world. The mental facts therefore supervene on the physical facts and any change in the former must be accompanied by a change in the latter.

Even if we separate the mental from the physical facts, change of epistemic states would still require factual change. Regardless of the truth of physicalism, assume a barrier between an agent and the environment in which he figures. The factual truths of the world now only describe the world beyond the agent, and the agent can change his internal states freely. The environment merely produces input that the agent evaluates to gather knowledge. Would it make sense for such an isolated agent to gain empirical knowledge of the outside world without factual change?

In his solitude, the agent could perhaps acquire knowledge of all *a priori* truths and deduce the validity of all logical tautologies. However, logical omniscience is already assumed by modeling knowledge as a modal operator since all possible worlds affirm the truth of logical necessities. The problem verificationism tries to tackle is the acquisition of empirical truths. For such *a posteriori* truths, however, our lone agent would require evidence in the form of new input. For the black box that is the world to provide new input, it must change. The agent may causally influence the change by conducting experiments,

but a factual change is always present.[3]

Fuhrmann (2014) admits that the receptivity of an agent to evidence, and subsequent revision of epistemic states as expressed by $<<$, is a matter of physical facts. Yet the potential acquisition of knowledge as modeled by $\langle K \rangle$ remains speculative and hypothetical. The claim $\langle K \rangle \phi$ asserts that given the agent's present disposition, he could potentially have a different epistemic state expressed by a relation that distinguishes ϕ worlds. Yet the actualization of this potency would have to be in a different world.

Consider the following case. Eve gives Adam a present to celebrate their anniversary. The present is meticulously wrapped in a box, leaving no hint as to what might be contained inside. Adam curiously asks about what is in the box. Eve urges him to unwrap the present instead. At this point Adam is in a state of ignorance, but, ever the optimist, he sees the glass as half full. His ignorance is potential knowledge. Eve could reveal the secret under further pressure, he could gain x-ray vision, or simply open the box. Any of those developments would update his epistemic state to an R with $R_{Adam} << R$ and $\forall w' \in W(wRw' \rightarrow w' \models \phi)$. None of those developments are true in the actual world w, however, and his epistemic state cannot change without one of them.

To acknowledge the demand for factual change for the acquisition of knowledge a posteriori, I propose a revision of Fuhrmann (2014) knowability interpretation. My solution uses a factual evolution of worlds to describe epistemic evolution and the crucial learning step.

3 Real knowledge potential

3.1 The framework of real knowledge potential

Real knowledge potential makes a distinction between hypothetical knowledge states K and real knowledge states E. Hypothetical knowledge states merely describe the capacity of a hypothetical agent to distinguish a world between different possible worlds. Actual knowledge states are instances of hypothetical states by some agent in some world. All truths about the constitution of agents in a world are part of its description. As a result, the real epistemic state of an agent in some world is uniquely specified by the facts of that world.

[3] This position is also presented by Heylen (2021) who argues that different knowledge states would have to follow from different investigations which often demand in turn alternative non-epistemic truths. The same point is defended more extensively in (Heylen and Carbonell, 2023).

This paper aims to capture the intuition of knowability as real knowledge potential. A proposition ϕ is hypothetically knowable by an agent in w with actual knowledge state E if there is a hypothetical state K such that $w \models K\phi$. In contrast, ϕ is really knowable if there is a world w' where the agent has a real knowledge state E' that instantiates a hypothetical K that renders ϕ hypothetically knowable.

The logic of real knowledge potential

To formalize real knowledge potential (RKP) I will use the following extension of modal logic.[4]

The language of real knowledge potential L_{RKP} is that of modal logic with the added symbols:

Symbol	Meaning
E	The actual knowledge state of the agent
K_i	A set Var of epistemic variables
$\forall$ and $\exists$	Quantifiers over hypothetical knowledge states
$=$	Equality over epistemic relations
$\langle K \rangle$	hypothetical knowledge potential of a hypothetical state
$\langle E \rangle$	hypothetical knowledge potential of a real state

The syntactic rules of L_{RKP} also allow for the following well-formed formulas. If ϕ is a well-formed formula then so are:

$$E\phi \mid K_i\phi \mid \forall K\phi \mid \exists K\phi \mid (E = K_i) \mid \langle K \rangle \phi \mid \langle E \rangle \phi$$

The frames that model the acquisition of knowledge as real knowledge potential are similar to the ones used by Fuhrmann (2014). The main difference is that the agents can have different epistemic states in different worlds. As mentioned before, the agent in a world can only have one real epistemic relation determined by the facts of the world. To differentiate it from hypothetical epistemic relations, I will refer to it as the instantiated relation. Limiting the analysis to one agent, to express the proposition "the agent in w instantiates R", I define an instantiating function Ins:

[4] The introduction of variables and quantifiers renders L_{RKP} more expressive than normal modal logic.

$$Ins : W \rightarrow ERL, \text{ where } ERL \subseteq \mathcal{P}(W^2) \qquad \text{(instantiating function)}$$

Hypothetical epistemic evolution can be imported from Fuhrmann (2014), only this time the evolution relation << depends on the reference world. A second instantiating function *Evo* models hypothetical epistemic evolution:

$$Evo : w \rightarrow ERL \times ERL$$

Hypothetical epistemic evolution will not play a part in RKP other than being compared to real epistemic evolution.

To define truth we once again begin with a valuation function assigning truth to all atomic propositions in P for every world in W.

$$V : W \times P \rightarrow \{0,1\}$$

The valuation function is then extended to $\bar{V}$ for all propositions. Since L_{RKP} includes epistemic variables, a variable assignment is required. A variable assignment s is a function $s : Var \rightarrow ERL$.

We can now define the truth of formula ϕ in a model M, a world w and variable assignment s, and write $w \models_M \phi[s]$ or $w \models \phi[s]$. The non-trivial cases are:

- $w \models K_i\phi[s]$ iff $w' \models \phi[s]$ for every w' such that $s(K_i)(w, w')$
- $w \models \langle K_i\rangle\phi[s]$ iff there is an $R \in ERL$ with $Evo(w)(s(K_i), R)$ and $w' \models \phi[s]$ for every w' such that $R(w, w')$
- $w \models E\phi[s]$ iff $w' \models \phi[s]$ for every w' such that $Ins(w)(w, w')$
- $w \models \langle E\rangle\phi[s]$ iff there is an $R \in ERL$ with $Evo(w)(Ins(w), R)$ and $w' \models \phi[s]$ for every w' such that $R(w, w')$
- $w \models \forall K\phi[s]$ iff for all $R \in ERL$, $w \models \phi[s(R/K)]$, where

$$s(R/K)(x) = \begin{cases} R & \text{if } x = K \\ s(x) & \text{otherwise} \end{cases}$$

- $w \models \exists K\phi[s]$ iff there exists an $R \in ERL$ such that $w \models \phi[s(R/K)]$
- $w \models (E = K)[s]$ iff $Ins(w) = s(K)$

A formula is true in a world w if it is true for all assignments s. A formula ϕ is valid in a frame S if is true for all worlds and all valuations, denoted as $\models \phi$.

Reinterpretations

With the aid of the above logical framework, the verification thesis can have the following reinterpretations.

$$\phi \rightarrow \langle E\rangle\phi \qquad \text{(hypothetical knowledge potential)}$$

This is Fuhrmann (2014)'s reinterpretation expressed in L_{RKP}. It stipulates that a proposition is knowable by the agent if his real knowledge state $Ins(w)$ can follow an epistemic evolution $Evo(Ins(w), R)$ to a new state R that discerns ϕ from $\neg\phi$ worlds.

Real knowledge potential will relinquish epistemic evolution for a change of worlds. In its simplest form, it uses the metaphysical possibility operator:

$$\phi \rightarrow \exists K(K\phi \wedge \Diamond(E = K)) \qquad \text{(naive real knowledge potential)}$$

Naive real knowledge potential makes a first step in demanding that the new epistemic state is really possible, that there is a world where the agent instantiates this state. In Adam's mind, it is epistemically possible that he could be God and be omniscient, yet it is metaphysically impossible for him to escape his ignorance this way. The naive interpretation excludes, at the very least, such claims of knowability.

Notably, the naive interpretation does not use the notion of epistemic evolution. Before venturing into improved versions of real knowledge potential, I will introduce the notion of real evolution to replace $\langle E\rangle$.

3.2 The evolution operator

The epistemic evolution operator $\langle K\rangle$ traverses the space of epistemic relations to find one that only pairs worlds that agree on ϕ. Should it be possible for the original world w to

only see ϕ worlds through the lens of an epistemic relation, then knowledge of ϕ would be achieved. To express this acquisition of knowledge as a path along possible worlds instead of possible relations a new operator is needed, the operator of real evolution, symbolized by $\blacklozenge$.

Similar to other modal operators, real evolution is expressed in the frame by a relation between worlds:

$$RealEvo \subseteq W \times W.$$

Since real evolution is a limited possibility operator, its semantics mirror that of metaphysical possibility:

- $w \models \blacklozenge\phi[s]$ iff $w' \models \phi[s]$ for some w' such that $RealEvo(w, w')$

Real evolution can be interpreted as the relation between worlds that can be part of a learning process. Starting from world w the agent can follow different lines of inquiry, collect evidence, and organize experiments, in general, the agent can augment his empirical knowledge. All these actions and events will alter reality and bring about a new world w'. In that case, we would write $RealEvo(w, w')$. The range of real evolution depends on our understanding of the ability to know. A narrow ability concept would only consult a few close worlds, while a wider one would be closer to metaphysical possibility.

Interestingly, if we follow the path of real evolution across possible worlds w and w', but only view the epistemic relations expressed by the instantiating function in these worlds, $Ins(w)$ and $Ins(w')$, then what we perceive would be an epistemic evolution. Epistemic evolution Evo can be redefined through Ins and $RealEvo$:

$Evo(R_1, R_2) \Leftrightarrow \exists w_1, w_2 \in W : Ins(w_1) = R_1 \wedge Ins(w_2) = R_2 \wedge RealEvo(w_1, w_2)$

What exactly is a learning process will be explored later when I present the model of epistemic updating. For now, real evolution is a possibility operator expressing the ability to know. It can thus replace metaphysical possibility in naive RKP:

$$\phi \rightarrow \exists K(K\phi \wedge \blacklozenge(E = K)) \qquad \text{(naive real knowledge potential)}$$

One crucial detail of real evolution is its interaction with equality regarding necessitation. For evolution to be fruitful it must lead to worlds where new epistemic relations are instantiated. The agent in these worlds would now have the ability to discern previously obscure truths. However, if equality is necessary throughout real evolution then $(E = K) \rightarrow \neg\blacklozenge\neg(E = K)$. As a result, any change through real evolution $RealEvo(w, w')$

would result in a world w' with the same epistemic relation, effectively blocking epistemic evolution. For real knowledge potential to express the acquisition of knowledge through a change in epistemic state, equality must be non-necessary.

$$\not\models (E = K) \rightarrow \neg\blacklozenge\neg(E = K) \qquad \text{(contingency of equality)}$$

3.3 Defending the evolution of epistemic states

In possible knowledge, knowledge is acquired by moving to a different world, one with a different epistemic horizon. Given the starting world a, the R epistemic horizon R_a is the set of worlds that can be seen from w through R.

$R_a := \{w \in W : R(a, w)\}$.

If $\forall w \in R_a(w \models \phi)$ or $\forall w \in R_a(w \not\models \phi)$ we say that this set of worlds agrees on ϕ and knowledge is achieved. If R_a agrees on ϕ we say that R can differentiate between ϕ and $\neg\phi$ worlds. In possible knowledge, we only seek an accessible neighbor whose epistemic horizon agrees on ϕ.

Knowability as epistemic potential not only demands a change in R_a, but a change in R itself. This is the principal distinction of Fuhrmann (2014)'s potential knowledge. In this paper, I try to envision this change in relations as the product of a real change in worlds, instead of a hypothetical evolution. First, however, I must argue that changing relations makes sense.

In Fuhrmann (2014)'s framework the entire world constellation is rigid. A new relation is the only way for the evaluation of knowledge to change. Fuhrmann (2014)'s defense of this evolution is the potential re-evaluation of evidence. Let us imagine that Adam is a detective in 1980 investigating a murder. One day he finds in his letterbox an envelope containing incriminating photographs of the victim's wife committing the crime. Adam is confident the woman is guilty as the only way for her to appear in a photograph with a smoking gun is by holding the gun while a camera points at her. In Adam's mind, the only epistemically possible worlds are ones where the wife did it.

Adam however is also a fan of science fiction films. He has therefore heard that in the future technology might be able to fabricate photo-realistic images rendering his evidence inconclusive. Perhaps the secret informant who delivered the photographs has access to such a technology. Adam thus believes it possible that he could have a different epistemic state in which the solution to the crime is obscure.

In a mirror example, Adam operates in 2030 when photographic evidence is useless for

crime solving and imagines living in 1980 when the life of an investigator was different. In either case, Adam imagines that his evaluation of epistemic possibility could be drastically different. It is in this possibility that Fuhrmann (2014)'s knowability resides.

Admittedly this is a fragile condition. Let us not forget that Adam has an alarming tendency to believe it to be epistemically possible that he could be God and thus omniscient. Real epistemic potential demands a stronger interpretation of knowledge potential. The new epistemic state must be expressed in a new world, one with factual differences justifying a different state of knowledge.

Real knowledge potential however has a harder time justifying the need to change relations. What is necessary for Fuhrmann (2014) who demands an immutable actual world, can now be achieved in the same fashion as naive knowability. Changing R_a suffices to express the epistemic differences of a detective in 1980 considering photographic material as conclusive evidence R_{1980}, and one in 2030 discarding them as facsimiles R_{2030}.

The trivial answer is we need it to bypass the Fitch paradox. As explained while discussing knowledge of other agents, $K_2(\phi \wedge \neg K_1\phi)$ is consistent while $K(\phi \wedge \neg K\phi)$ is not. Yet in the context of real change and the migration to a new world, the change of epistemic relation has a bizarre side effect. Similar to Edgington's demand of two-dimensional frames for knowledge of actuality, it provides an access to other worlds that may seem absurd.

By moving to a new world, we change the set of epistemically accessible worlds from R_w to $R_{w'}$. The concurrent change of relation further alters this horizon to $R'_{w'}$, where R' is the new epistemic state. At the same time, the set of epistemically accessible states in the original world w is now R'_w, different from R_w. Assigning an epistemic relation to an agent describes not only the worlds he sees as possible but also the worlds he would consider possible had his own world been different but somehow miraculously his epistemic state remained the same. Crucially, as discussed in the previous section about the contingency of equality, this hypothetical set R'_w must differ from R_w to satisfy naive RKP and express acquisition of knowledge. Since Fuhrmann (2014) never changes worlds, this access to different worlds remains salient and inert. It is, however, the driving force of real knowledge potential and thus needs to be defended.

3.4 The power to differentiate worlds

Returning to the first thought experiment of Adam in the cave of ignorance, we can examine two distinct possible worlds. In world c Adam is in the cave of ignorance and his epistemic state is described by relation I. In the second world v Adam is in the valley of omniscience and his state is modeled by the relation O. The proposition in question is p "Adam is in the cave of ignorance" which is only true in world c. Relations I and O define the following horizons (Figure 1).

$I_c = \{c, v\}, I_v = \{v\}, O_c = \{c\}, O_v = \{v\}$

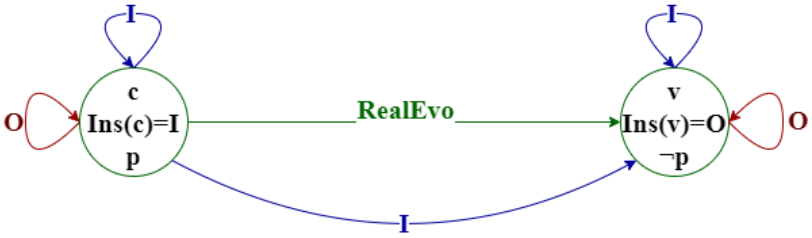


Figure 1: The model of the cave of ignorance and the valley of omniscience

Let us assume that starting from c a real evolution is possible that will result in world v, $RealEvo(c, v)$, that Adam can travel to the valley of omniscience and gain state O. When Adam enters the valley of omniscience and acquires O he will be able to say "I now know $\neg p$ and I know that if I were in the cave of ignorance I would not know p despite it being true". Here the first two uses of "I know" refer to O while the last one refers to I. If we could identify Adam based on his epistemic state, we would say that in world c the omniscient Adam knows p and he also knows that the oblivious Adam does not know p.

However, the omniscient Adam only exists really in v. In c the omniscient Adam is merely a hypothetical agent. One that the real Adam in c who is oblivious can transform into following a real evolution. Naive real knowledge potential states precisely that. Adam can evolve his epistemic state, by moving to a new world, into a new one that would grant him knowledge if he hypothetically possessed it in the original world.

The best way to understand this expression of a hypothetical counterfactual is as a power. When Adam acquires O he gains a new ability to discern the identity of worlds. Should he have this power while in the cave of ignorance, he would be able to know the truth. He would also know that an agent with an inferior state would live in ignorance. Take the example of vision. Imagine Adam is born blind and thus has no clue of the different colors that adorn his world. As a child, he stood in the middle of his room

wondering what the color of his ceiling was. It was blue but he did not know it. Years later, through some miraculous procedure, Adam acquires vision. With this new power, he can enter his old room and gaze at the color of the ceiling. Yet the ceiling could have been repainted in the years that passed. However, had Adam possessed the ability to see while staring at the ceiling as a kid, he would have been able to know that it was in fact blue, as well as that had he been blind he could not possibly know that.

Unfortunately, the epistemic state describing Adam in any world is determined by the facts of the world, hence Adam could not possess any state different from the one he has. In c Adam could not but be oblivious. A world c' that is identical to c in all truths but the epistemic relation instantiated by Adam must be inconsistent.

3.5 Improving real knowledge potential - Knowledge acquisition

Naive knowledge potential has glaring flaws. As evident from the example of sight, acquiring a new state does not entail knowledge. Adam could gain vision and still not know what colour his childhood room was. This oversight is present in the formalization as it does not refer to knowledge, only to the instantiation of a new state. Moreover, naive knowledge potential remains too close to Fuhrmann (2014)'s knowledge potential in demanding hypothetical epistemic states. The hypothetical sighted Adam could know what the real blind Adam does not. To better capture the acquisition of knowledge, RKP must delineate a change of worlds that ends in knowledge.

To that end, the first improvement is to shift the hypothetical step from the original world and the agent, to the new world and the proposition. Instead of requiring the knowledge of a hypothetical agent in the original world, we demand the knowledge of the real agent of the new world of a hypothetical proposition.

Adam has recently acquired vision and visits his childhood bedroom. If we express the Fitch sentence "the ceiling is blue and I do not know it" as $\phi \wedge \neg E\phi$, then $E(\phi \wedge \neg E\phi)$ is inconsistent. Naive knowledge potential renders the outer E hypothetical and demands $w \models K(\phi \wedge \neg E\phi)$. The next step is demanding that in the evolved world there is also knowledge, by blocking the Fitch sentence by changing the inner E to a hypothetical. Following the epistemic evolution and the acquisition of sight, Adam can state "The ceiling is blue and if I were still blind I would not know it" formalized as $w' \models E(\phi \wedge \neg K\phi)[s(Ins(w)/K)]$.

Knowledge belongs to the real agent yet is hypothetical. Proposition ϕ: $p \wedge \neg Ep$ in the original world, and ψ: $p \wedge \neg Kp$ in the evolved world, are intentionally different but have

the same reference, the epistemic state before evolution. I will refer to ψ as the epistemic projection of ϕ on world w. Epistemic projections can be formalized using an operator $Pr(w)$, semantically defined in the following way.

- $a \models Pr(w)\phi[s]$ iff $a \models \phi(K/E)[s(Ins(w)/K)]$[5]

Proposition $\phi(K/E)$ is the product of substituting all appearances of E by the epistemic variable K in ϕ and can be defined recursively.

Given ϕ, its projection is a proposition where every reference to the actual epistemic state E is replaced by a hypothetical variable K whose assignment is fixed to the instantiated state of the world w targeted by the projection. Epistemic projection allows propositions to have stable references to immutable epistemic relations throughout epistemic evolution. To express knowledge in the evolved world the new demand is $e \models E(Pr(o)\phi)$ where o is the original world and e the evolved world.

The epistemic projection shares many similarities with the actuality operator employed in Edgington (1985)'s proposal. It enables the blocking of the Fitch sentence by way of reference to facts of non-actual worlds. Yet an epistemic projection does not require full access to a non-actual world, only to its instantiated epistemic state. While having *de re* access to other worlds is unpalatable, *de re* access to limited facts of other worlds, like the epistemic state of the agent in them, is perhaps tolerable.

Real knowledge potential improves upon the naive version by including this request for epistemic evolution. Here epistemic projections are formalized with a *de re* reference to the original epistemic state.

$$\exists K_1((E = K_1) \wedge \blacklozenge E(\phi(K_1/E))) \qquad \text{(knowledge acquisition - KA)}$$

In combination with naive RKP, KA results in the following formulation of verification:

$$\phi \rightarrow \exists K_1 \exists K_2((E = K_1) \wedge K_2\phi \wedge \blacklozenge((E = K_2) \wedge E(\phi(K_1/E)))) \qquad \text{(real knowledge potential)}$$

3.6 Evaluating the real knowledge potential reinterpretation

The first step in evaluating whether a suggested reinterpretation fulfills the verificationism intuition is subjecting it to the necessary conditions it must satisfy. Is it factive? Does it apply to all propositions or does it exclude important ones? Finally, does it overcome omniscience only to lead to possible omniscience? The question of restrictions is the easiest

[5] $Pr(w)$ and world variables will not be part of the language, they are only used for comparison purposes

to answer as RKP can be applied to all well-formed formulas. Factivity and omniscience require a closer examination.

Factivity

If the verification intuition of "all truths are knowable" is to be translated as "all truths have real knowledge potential" then the following conditionals are to be considered the same:

$$\phi \rightarrow \mathit{Knowable} \qquad \text{(verification)}$$

$$\phi \rightarrow \exists K_1 \exists K_2((E = K_1) \wedge K_2\phi \wedge \blacklozenge((E = K_2) \wedge E(\phi(K_1/E)))) \qquad \text{(RKP)}$$

Factivity is the statement that only true propositions are knowable:

$$\mathit{Knowable} \rightarrow \phi \qquad \text{(factivity)}$$

In the context of RKP, factivity requires the validity of the following conditional:

$$\exists K_1 \exists K_2((E = K_1) \wedge K_2\phi \wedge \blacklozenge((E = K_2) \wedge E(\phi(K_1/E)))) \rightarrow \phi \qquad \text{(factivity of RKP)}$$

The above is trivially proven since part of RKP is $\exists K_2(K_2\phi)$. The only requirement is limiting the frame to reflexive epistemic relations, thus ensuring factivity of knowledge:

$$\forall K(K\phi \rightarrow \phi) \qquad \text{(factivity of knowledge)}$$

Reflexivity is the following frame condition:

$$ERL \subseteq \{R \in \mathcal{P}(W^2) : \forall u \in W :< u, u >\in R\} \qquad \text{(reflexivity)}$$

Theorem 1. *(naive) RKP is factive.*

Proof.

Let RKP be true world w and proposition ϕ:

$$w \models \exists K_1 \exists K_2((E = K_1) \wedge K_2\phi \wedge \blacklozenge((E = K_2) \wedge E(\phi(K_1/E)))) \qquad \text{(RKP)}$$

According to the semantic definition of $\exists K$, there should exist relations R_1 and R_2 with $R_1, R_2 \in ERL$ such that:

$$w \models ((E = K_1) \wedge K_2\phi \wedge \blacklozenge((E = K_2) \wedge E(\phi(K_1/E)))) \; [s(R_1/K_1, R_2/K_2)]$$

Furthermore, from the semantic definition of $\wedge$ it should be true that:

$$w \models K_2\phi[(s(R_1/K_1, R_2/K_2)]$$

which is true if and only if $w' \models \phi$ for every world w' such that $R_2(w, w')$. From reflexivity, $R_2(w, w)$ and thus $w \models \phi$. □

A possible critique of the factivity of RKP becomes apparent when one separates the formalization into its constituents.

On the one hand naive RKP:

$$\exists K_2(K_2\phi \wedge \blacklozenge(E = K_2)) \qquad \text{(naive RKP)}$$

and on the other the knowledge acquisition component:

$$\exists K_1((E = K_1) \wedge \blacklozenge E(\phi(K_1/E))) \qquad \text{(KA)}$$

As evident from the above proof, the factivity of RKP depends entirely on the naive RKP. The further condition of KA cannot sustain factivity on its own. It is possible that KA is satisfied for a false proposition. For a counter example consider a model describing a present with three possible worlds, w_{ce} where the box is closed yet empty, w_{cr} where the box is closed and contains a ring, and w_{or} where the box is open and the ring revealed. The proposition in question is p : "there is a ring in the box". For simplicity, the agent instantiates the same relation in all worlds:

$Ins(w_{oe}) = Ins(w_{ce}) = Ins(w_{or}) = R$,

$R_{w_{ce}} = R_{w_{cr}} = \{w_{ce}, w_{cr}\}$, $R_{w_{or}} = \{w_{or}\}$

Moreover, assume real evolution is broad enough to allow $RealEvo(w_{ce}, w_{or})$, then the following are true:

$w_{or} \models Ep$

$w_{or} \models Ep(K/E)[s(R/K)]$

$w_{ce} \models \blacklozenge Ep(K/E)[s(R/K)]$

$w_{ce} \models \exists K_1((E = K_1) \wedge \blacklozenge Ep(K_1/E))$

$w_{ce} \models \neg p \wedge \exists K_1((E = K_1) \wedge \blacklozenge E(p(K_1/E)))$

As a result, a reinterpretation of knowability that relies exclusively on the knowledge acquisition component of RKP, would be non-factive. The naive RKP is indispensable in maintaining factivity by anchoring the possible epistemic relations accessible through real evolution to the original world.

Omniscience

The last test for RKP is potential omniscience. At the heart of real knowledge potential lies the real evolution represented by ♦. If a proposition has real knowledge potential then there exists a novel world, reachable through real evolution, in which the agent gains the desired epistemic state. The verification thesis is reinterpreted as the postulate that all propositions satisfy RKP, that for every ϕ there is such a novel world. However, it does not prevent all propositions from being known in the same world.

As explained in the case of Adam in the valley of knowledge, omniscience can be interpreted as the power to differentiate between worlds and identify the actual one, whatever it may be. In the form of epistemic relations, omniscience is modeled by the identity relation Id: $(w_1, w_2) \in Id \Leftrightarrow w_1 = w_2$. If a world o is possible in which the agent instantiates the identity relation, $Ins(o) = Id$, and furthermore o is reachable from w through real evolution, $RealEvo(w, o)$, then the above scenario is possible. In the pursuit of knowledge, the agent can arrive at a single world, o in which everything is known.

To avoid rendering this possibility inevitable, real evolution, which has so far remained obscure, must be carefully curated to deny access to such a world. The model of epistemic updating presented in the last chapter will give an example of real evolution that avoids potential omniscience.

4 Knowablity in a temporal world structure

4.1 Learning as a process

RKP tries to improve upon Fuhrmann (2014)'s framework by acknowledging that any change in epistemic state must be accounted by facts of the world. If Adam knows the truth about the cave of ignorance it is because he walked out and into the light. If he knows the color of his childhood bedroom it is because he gained vision. Real evolution models this process of acquiring knowledge through changes in the facts of the world. It is therefore the catalyst that enables RKP to go beyond a simple search of an improved epistemic relation, to a possible world where this relation is really acquired by the agent.

Admittedly, Fuhrmann (2014)'s static framework can overcome this critique by adapting the propositions to remove tensed elements. Adam can be in the cave of ignorance at moment t_1, while at t_2 he is in the valley of knowledge. He can be blind on day d_1 and on day d_2 gain vision. All the above could be ontic truths ingrained in the world in question, allowing the K operator to implicitly quantify over time. Even the evolution of epistemic

states from R_1 to R_2 could be represented by a fact in the domain of P. The factual changes required for learning new propositions could be aggregated and packaged in a single world. The result would be a crude model that relies too heavily on the semantic translation of the propositions. Real knowledge potential allows for a refined model of the learning process that illustrates the factual changes needed in order to acquire knowledge.

For this reason, while any possibility operator can function as real evolution, I propose a solution that interacts with a temporal ordering of the worlds. The idea is that learning as a process establishes a temporality, a journey towards knowledge, with numerous twists and turns, and alternative paths that may or may not be followed. A temporal structure would do justice to the intricacies of learning. Possible lines of inquiry, the completion of which is for Schlöder (2019) a fact of the world, will be re-imagined as paths through possible worlds.

To that end, I will first enrich the structures of the RKP language to organize the possible worlds into timelines. With the aid of this partial ordering, the demands of real change for knowledge gain can be further investigated for their soundness and feasibility.

4.2 Temporal structures

The first step in enriching L_{RKP} to get L_{RKPT} with temporal operators, is introducing a partial ordering over the possible worlds in W. This temporal ordering will be represented by $\preceq$. When $w_1 \preceq w_2$ then world w_1 precedes w_2 or is w_2. Two operators, B and A standing for "before" and "after", will be making use of the temporal ordering. The semantics now also include the following rules.

- $w \models B\phi[s]$ iff there exists a w' such that:
  - $w' \preceq w$
  - $w' \neq w$
  - for all w'': $((w' \preceq w'') \wedge (w'' \preceq w)) \rightarrow ((w' = w'') \vee (w = w''))$
  - $w' \models \phi[s]$
- $w \models A\phi[s]$ iff there exists a w' such that:
  - $w \preceq w'$
  - $w' \neq w$

– for all w'': $((w \preceq w'') \wedge (w'' \preceq w')) \rightarrow ((w' = w'') \vee (w = w''))$

– $w' \models \phi[s]$

B and A evaluate truth at a singular time step away from the original world. For ease of use, the first three conditions will constitute the time step relation $\prec$. More operators can be defined by removing some of those conditions. I will denote by * the omission of the second condition and by $^+$ the omission of the third. For example, B^{*+} allows for any past or present world.

Temporal frame restrictions

For $\preceq$ to describe a temporal ordering, it must be a partial ordering of the worlds and thus reflexive, antisymmetric, and transitive:

- $\forall w \in W : w \preceq w$ (reflexivity)
- $(w_1 \preceq w_2) \wedge (w_1 \preceq w_2) \Rightarrow (w_1 = w_2)$ (antisymmetry)
- $((w_1 \preceq w_2) \wedge (w_2 \preceq w_3)) \Rightarrow (w_1 \preceq w_3)$ (transitivity)

Moreover, $\preceq$ has no infimum or supremum, describing how time has neither a beginning nor an end:

- $\forall w \exists w'((w \neq w') \wedge (w \preceq w'))$ (no supremum)
- $\forall w \exists w'((w \neq w') \wedge (w' \preceq w))$ (no infimum)

Furthermore, two timelines can never converge, as every world must have a unique past.

- $\forall w_1 \forall w_2 \forall w_3(((w_1 \preceq w_3) \wedge (w_2 \preceq w_3)) \rightarrow ((w_1 \preceq w_2) \vee (w_2 \preceq w_1)))$ (unique past)

A similar restriction can be made for branching timelines leading to a deterministic framework.

- $\forall w_1 \forall w_2 \forall w_3(((w_3 \preceq w_1) \wedge (w_3 \preceq w_2)) \rightarrow ((w_1 \preceq w_2) \vee (w_2 \preceq w_1)))$ (unique future)

These two conditions organize the possible worlds into timelines without enforcing connectivity. A timeline is a sequence of worlds $w_1, w_2, w_3, ...$ where:

$\forall i, j \in \mathbb{N}((i \leq j) \rightarrow (w_i \preceq w_j))$

Connectivity would require that for any two worlds $v, u \in W$, $v \preceq u$ or $u \preceq v$, thus collapsing the partial order into a total one. In the absence of connectivity, the frame allows for alternate possible timelines.

In the deterministic model delineated by the above frame conditions, the worlds of every timeline can be organized into equivalence classes $[a]$:

$[a] = \{w \in W : (w \preceq a) \vee (a \preceq w)\}$.

Finally, to talk about truths in a timeline, the following notation will be used:

$[w] \models \phi$ if and only if for all w' in $[w]$, $w' \models \phi$

Eternal truths

The eternal propositions EP are the propositions that are true in all worlds of any timeline:

$EP := \{\phi \in FORM : \forall w : (w \models \phi) \rightarrow [w] \models \phi\}$

$EP_w := \{\phi \in EP : [w] \models \phi\}$

Timelines and eternal truths can be used to create a static model describing tenseless propositions. In a static model, all uses of $w \models \phi$ are replaced by $[w] \models \phi$, and ϕ are limited to eternal propositions in EP. As mentioned in the previous section, Fuhrmann (2014)'s potential knowledge could perhaps operate in such a framework to overcome the criticism of factual immutability in the face of epistemic change. Nevertheless, the temporal structures better illustrate the dynamic process of learning, and will therefore be part of L_{RKPT} moving forward.

4.3 Improving real knowledge potential - Knowledge of past truths

Using temporal operators, the RKP reinterpretation can be improved further to be true in more models. A problem of RKP, and the KA component in particular, is that it demands the truth of ϕ in the new world. Intuitively that seems like an innocent demand. After all, knowability must be factive, and thus moving to a world where ϕ is no longer true in an effort to know it seems contradictory. However, a problem arises from propositions that uniquely identify worlds.

Returning to the example of section 2.4, the proposition in question was p: "Adam is in the cave of ignorance". Proposition p is only true in world c describing the cave. As a result, while p can be knowable in the naive RKP sense, it cannot be known in the KA sense. KA demands the truth of p in the world resulting of real evolution. If p is only true in c, then the sole candidate for real evolution is the original world. Yet in the original world, the instantiated relation is one of ignorance.

However, propositions that uniquely identify worlds are in stark opposition to the project of verificationism. Knowing one such proposition would entail omniscience. They can thus easily be dismissed. Unfortunately, the problem persists without them. Consider a case in which a specific fact of the world prevents the knowledge of a proposition. One example is of a box containing a sapphire. We examine the following propositions b: "the box is open" and s "there is a sapphire in the box". Let us imagine that through some quantum magic, while the box is closed there is no way of knowing what lies inside the box.[6] A random physical process can generate a sapphire or any other precious gemstone. The model includes, among others, the following worlds:

- $w_0 \models s \wedge \neg b$
- $w_1 \models \neg s \wedge \neg b$
- $w_2 \models s \wedge b$
- $w_3 \models \neg s \wedge b$

Moreover, no one in w_0 can know what is in the box without opening it:

- $w_0 \models \Box(\neg b \rightarrow \neg Es)$

The relevant epistemic relations are R_1 which cannot differentiate worlds based on the gemstone inside the box, and R_2 which can. R_1 is instantiated at $\neg b$ worlds while R_2 at b worlds.

In this model the proposition $\phi : s \wedge \neg b$ does not uniquely identify a world, as we can imagine a similar experiment in world w_0' in which only the color of the box changes, yet the truth of ϕ remains. However, ϕ cannot be knowable by the KA component of RKP since $E(\phi(K/E)) = E\phi = E(s \wedge \neg b)$. From factivity and distribution of conjunction we arrive at $Es \wedge \neg b$ which contradicts $\neg b \rightarrow \neg Es$

[6] Heylen (2024) presents a similar example of a tomb that may or may not contain a buried pharaoh. The only way to know is by breaking the door and looking inside, at which point the proposition "the door is closed and there is a buried pharaoh in the tomb" is no longer true.

The only way to achieve KA is to break the necessity of $\neg b \rightarrow \neg Es$. If $\neg b \rightarrow \neg Es$ is not true in the model, but merely in some worlds, then $\phi \wedge (\neg b \rightarrow \neg Es)$ can be KA knowable. It would still be a bizarre conclusion since we moved to a world where we know the content of the box without opening it and that opening it was the only way to know its content. In essence, we are abusing the weak conditional $\neg b \rightarrow \neg Es$ by traveling to a world that rejects it, instead of respecting the stronger $\Box(\neg b \rightarrow \neg Es)$ by limiting the reach of real evolution within that of metaphysical necessity.

Such propositions that cannot be known without specific factual changes are not entirely hypothetical, or necessarily linked to quantum uncertainties. Schlöder (2019) gives the example of measuring the pressure of a bike. Since any measurement is bound to release some air, the knowledge acquired from it no longer represents the past truth.

The new temporal operators offer a simple solution. Instead of demanding knowledge of $\phi(K/E)$ we can be content with knowing that it has been true, $B\phi(K/E)$, resulting in the following improvement of the knowledge acquisition:

$$\exists K_1((E = K_1) \wedge \blacklozenge E(B\phi(K_1/E))) \qquad \text{(knowledge acquisition of past truths (KAP))}$$

Now, real evolution can lead to a world where the box is opened and the sapphire seen, to a new epistemic relation that only sees sapphire worlds, and to knowledge that at some point in the past, the box was closed, its contents were unknown and opening it was the only way to see what is inside.

4.4 Real evolution and ability concepts

The other major contribution of temporal structures to RKP is illuminating the differences between the various ability concepts real evolution can portray. In what sense is the verification thesis that "all truths are knowable" to be understood? Will they be known in the future, or only in a similar albeit not identical world? As mentioned before, the reach of real evolution describes how narrow or broad the notion of ability is. I will draw three main categories based on how real evolution interacts with timelines, metaphysical ability, epistemic ability, and actual ability. Metaphysical ability refers to broad ability concepts that allow for vastly different worlds and timelines, epistemic ability limits accessible worlds according to the information at hand, and lastly, actual ability demands present or future knowledge in the actual timeline.

The various interpretations often leave this point obscure. Possible knowledge relies on the metaphysical possibility of a world where there is knowledge. If such a world exists, the knowability postulate is satisfied. Similarly, Edgington (1985)'s proposal seems at first glance to demand mere metaphysical possibility, yet the attempts to salvage her interpretation often employ descriptions of the actual situation or its causal history which seem to suggest an epistemic possibility. In the same vein, Schlöder (2019) enlists counterfactuals to refer to close-by worlds, though it is unclear whether some universal categories define inter-world affinity or if their similarities should be understood in an epistemic sense. Lastly, Fuhrmann (2014)'s proposal of potential knowledge, which has been the point of departure for real knowledge potential, incorporates a notion of susceptibility to evidence, which seems to lean closer to an epistemic ability.

RKP can express different verification postulates by appropriately abridging the domain of real evolution. Real evolution, as its name suggests, describes a change in reality, in factual truths, and depicts a transition to a new world. So far no boundaries or restraints have been placed upon it. Depending on the restriction, RKP will describe a metaphysical, epistemic, or actual ability concept.

Metaphysical ability

The first option is allowing real evolution to range over all possible worlds. Such a position would only enforce the weakest restriction that real evolution should respect metaphysical possibility:

$RealEvo \subseteq N$ (metaphysical ability)

While the easiest to satisfy, it expresses a mundane claim of knowability. It is metaphysically possible that human beings could possess the ability of echolocation, yet that is hardly what we have in mind when claiming that a myopic individual could know where he left his glasses.

Actual ability

On the other extreme lies actual ability. A proposition is actually knowable if and only if it will someday come to be known. This is expressed by the frame condition:

$RealEvo(w, v) \Rightarrow w \preceq v$ (actual ability)

This position states that the only world changes relevant to the evaluation of knowabil-

ity are ones that respect the progress of time. A claim of knowability under this restriction requires that for every truth, there is a future world where it is known. While not equivalent to potential omniscience, as there is no demand for a future where all truths are known, the claim nonetheless appears untenable. On the one hand, we can claim with relative certainty that there is at least one decidable truth never to be known by man. Problems like the enumeration of all prime numbers will probably outlive human society. On the other hand, just because the astronomer will not look at the sky during a comet's passing, does not mean he cannot. The postulate seems to collapse ability into actuality and should therefore be avoided. Still, without delving into debates of determinism, this criticism can be circumvented by rejecting the unique future frame condition of $\preceq$ resulting in an indeterministic model of time. For the aim of this paper, it suffices to show that the notion of real evolution can formulate even this narrower understanding of knowability.

Epistemic ability

Lastly, epistemic ability stands in the middle of extremes. In this option, real evolution can look to worlds outside the actual timeline, yet without allowing all metaphysically possible world transitions. In this sense, knowability is closely related to knowledge and what worlds the agent deems possible, given evidence or lack thereof. Two candidate restrictions are:

- $RealEvo(w_1, w_2)$ only if there is a w_3 such that $w_3 \preceq w_2$ and $Ins(a)(w_1, w_3)$ where a is the original world.
- $RealEvo(w_1, w_2)$ only if there is a w_3 such that $w_3 \preceq w_2$ and $Ins(w_1)(w_1, w_3)$.

The first option closely resembles Fuhrmann (2014)'s interpretation as it only consults the epistemic state of the agent in the original world. The second demands the continuous reevaluation of acceptable world changes following the dynamic process of learning. As the instantiated relation changes through real evolution, so does real evolution itself. For now, human efforts in the sciences are limited by the restraints of computability and decidability, as well as the existing means of experimentation. Perhaps in the future new methods of uncovering truths will be discovered, completely changing the possible directions of research. For these reasons, I prefer the second definition of epistemic real evolution.

4.5 Vulnerabilities of real evolution - Knowledge of eternal truths

While ambitious in theory, the more liberal restraints of *RealEvo* create various issues concerning truth preservation. As real evolution moves from w_1 beyond the confines of $\preceq$ to w_2, it reaches a new timeline $[w_2]$ and thus a different set of eternal truths. As a result, Some propositions true in w_1 are false in w_2 and any world temporally connected to w_2. Therefore, such propositions cannot be KAP knowable by following this evolution of worlds.

Theorem 2. *For every formula ϕ that is KAP knowable at w through real evolution to world v, $[v] \neq [w]$, but is not future knowledge (ϕ not KAP knowable through any $[w]_t$), there is a formula ψ not KAP knowable in w through v.*

Proof.

Assume ϕ is the proposition whose knowability is in question. Moreover, assume that ϕ is one of the truths that will never actually be known in the future. Formally for every w' with $w \preceq w'$, KAP is not satisfied:

$$w' \not\models E(B^{*+}\phi(K_1/E))[s(Ins(w)/K_1)] \qquad \text{(not future knowledge)}$$

Moreover, assume that RKP is true and ϕ is nonetheless knowable, meaning that there is a v with $RealEvo(w, v)$. It should fulfil the following conditions:

$$v \models E(B\phi(K_1/E))[s(Ins(w)/K_1)] \qquad \text{(KAP)}$$

$$w \not\preceq v \text{ thus, } [w] \neq [v] \qquad \text{(different future)}$$

Furthermore, we can safely assume that timeline $[v]$ is not a mirror of $[w]$ meaning that $EP_w \neq EP_v$. If $EP_w = EP_v$ then any knowledge gained through $RealEvo(w, v)$ should also be obtainable via some $RealEvo(w, w')$ where $w' \in [w]$ and $Ins(w') = Ins(v)$. Yet it is already presupposed that ϕ will not be known in the future.

There is therefore a proposition $q \in EP_w$ that is not in EP_v. Thus for all v' such that $v' \preceq v$ it holds that:

$v' \not\models q$.

As a result:

$v \not\models Bq$.

From reflexivity of $Ins(v)$ it follows:

$v \not\models EBq$.

Lastly, the distribution of conjunction on E entails:

$v \not\models E(B(\phi(K_1/E) \wedge q))[s(Ins(w)/K_1)]$.

However, $w \models (\phi \wedge q)$, thus $(\phi \wedge q)$ is not KAP knowable through $RealEvo(w, v)$

□

Whenever *RealEvo* necessarily changes timeless, a loss of eternal truths occurs. Propositions that satisfy KAP through $RealEvo(w, v)$ can be false in w (loss of factivity), and others can be true in w but no longer knowable (loss of knowability). For the RKP postulate to be maintained, a different evolution $RealEvo(w, u)$ is required for the proposition $\phi \wedge q$. Once more, however, *RealEvo* must venture beyond the limits of $\preceq$ leading to the discovery of a new eternal truth q' that will not be knowable through it.

The above process can be repeated to narrow down possible paths to knowledge. Starting from any proposition ϕ that will never be known in the future, we continuously arrive at new worlds w' with $RealEvo(w, w')$ that grant KAP, only to find a new eternal truth q such that $[w] \models q$ and $[w'] \not\models q$. Finally, augmenting ϕ to $\phi \wedge q$ creates a new true proposition that will never be known in the future, and the vicious circle continues, only this time with fewer options for real evolution. If the number of eternal truths is finite, this process will eventually construct a proposition that can only be knowable in the actual future, leading to a contradiction.

Strengthening the requirements by demanding that *RealEvo* respects the epistemic state of the agent does not overcome this issue. It merely protects the agent from knowing that the path to knowledge he foresees is incompatible with the facts of his world. The RKP thesis is thus forced to accept that if there are truths that will never actually be known, the model must consist of infinite alternate timelines and infinite eternal truths.

4.6 Revisiting factivity - Indeterministic model

As concluded in the evaluation of factivity, naive RKP is solely responsible for anchoring knowability to truths. The KA component, which crucially aggregates knowledge after every evolution, cannot guarantee that the truths it discovers are true in the original world. The temporal augmentation of the language can also shed light on the difficulties of preserving factivity while creating a concept of knowability.

The essential characteristic of real evolution is the use of world changes to model knowledge acquisition. As in every interpretation of knowability that relies on a possibility

operator, factivity depends on the reversibility of this process, namely *RealEvo* for RKP. The very problem that Edgington (1985) solves with a reference to the actual world, naive RKP solves with a reference to the actual epistemic relation. Perhaps the introduction of the B operator can solve the same problem for KAP. If so, then naive RKP can be discarded and KAP alone should take its place.

Unfortunately, this is not the case. KAP demands that real evolution brings about a tomorrow where some truth is known about yesterday. Yet this tomorrow's yesterday is not today. To decipher this cryptic sentence we need to unravel the definitions of KAP and B. If KAP were factive and w a world that satisfies it for some ϕ, we should be able to derive $w \models \phi$:

Attempted Proof:

$w \models \exists K_1((E = K_1) \wedge \blacklozenge E(B\phi(K_1/E)))$ (KAP)

$\Rightarrow w \models \blacklozenge E(B\phi(K_1/E))[s(Ins(w)/K_1)])$

$\Rightarrow \exists v \in W : RealEvo(w, v)$ and $v \models E(B\phi(K_1/E))[s(Ins(w)/K_1)])$

Moreover, from the second conjunct:

$v \models E(B\phi(K_1/E))[s(Ins(w)/K_1)])$

$\Rightarrow \forall v' \in Ins(v)_v : v' \models B\phi(K_1/E)[s(Ins(w)/K_1)])$

$Ins(v)$ is reflexive thus:

$v \models B\phi(K_1/E)[s(Ins(w)/K_1)])$

$\exists u \in W : u \prec v$ and $u \models \phi(K_1/E)[s(Ins(w)/K_1)])$

The search for factivity can go no further as there is no guarantee that u and w are identical or even that $u \models \phi \Rightarrow w \models \phi$. To reintroduce factivity we must take steps to enforce one of the above.

Option 1: $u \models \phi \Rightarrow w \models \phi$

The first option is ensuring $u \models \phi \Rightarrow w \models \phi$ for all ϕ. Informally this condition necessitates that real evolution only sees timelines with equivalent pasts. The following model condition can realize this requirement:

$RealEvo(w, v) \Rightarrow \forall u \in W : ((u \prec v) \Rightarrow (u \models \phi \Rightarrow w \models \phi))$ (forced factivity)

The question that emerges is how can two timelines with the same histories be meaningfully different. Keep in mind that we are still operating in a deterministic framework. Further on, the nature of ϕ has intentionally remained opaque. If it includes eternal truths then the timelines themselves must be extensionally identical. At the same time, if *RealEvo*

is meant to model what the agent considers epistemically possible futures, then the above demand entails that his epistemic horizon is comprised exclusively of mirrored timelines. Once more omniscience lurks around the corner.

The only way to implement forced factivity is in a restricted pool of propositions. Eternal truths and truths about the future must be excluded, perhaps even truths that necessitate future events. Such an endeavor is beyond the scope of this paper which refrains from any restrictions.

Option 2: $u = w$

This solution demands that real evolution can only reach worlds whose past is the original world. Real evolution would then express an actual ability concept, as evident from the following formal frame restriction:

$RealEvo \subseteq \{< v, u >: (v \prec u)\}$

It becomes apparent that this solution describes knowability as future knowledge. Only in such an extremely narrow ability concept, can factivity be safeguarded. The only way forward in this approach is to relinquish determinism.

KAP can be made factive by employing an indeterministic model of time. As mentioned before, by rejecting the unique future frame condition, real evolution can depict the knowability of propositions that will never actually be known, while being limited to the $\prec$ relation. Keep in mind that the definition of a timeline as an equivalence class is no longer applicable since worlds can belong to multiple timelines. Timelines must be reimagined as maximal absolutely ordered subsets of $\preceq$. Moreover, worlds cannot evaluate eternal truths. In the end, the results would be no different than those of forced factivity. Embracing indeterminism merely offers a natural restriction of eternal propositions by the very construction of the model.

Finally, knowability as knowledge in some future is still a very narrow ability concept. Imagine Adam is presented with an envelope containing a letter from Eve. Eve has recently been unfaithful causing a rift in their marriage. The letter either contains an apology or a request for divorce. Adam, afraid of confronting the truth burns the letter. Before doing so, however, there was a possible future where Adam read the letter instead. Now it is too late, the envelope is ashes, and any truth it contained is lost forever. In no possible future can Adam recover the letter, or discover the truth in other means (the traumatic separation leaves Eve with permanent memory loss). A KAP model with indeterministic time would concede that the contents of the letter are no longer knowable. There is however

an alternative timeline, one that does not share the same past, where this truth is known. Depending on the range of the ability concept, verificationism can allow for forever-lost truths.

5 Epistemic updating - A dynamic empirical learning process

5.1 Real evolution as empirical announcements

In the last part of this paper, I will present a model of an empirical learning process using real evolution. The model recreates a dynamic learning process through a series of announcements depicting empirical discoveries. The problem of omniscience will eventually be brought to the forefront prompting new ways to avoid it. The main sources of inspiration are Arbitrary Public Announcement Logic (APAL) developed by Balbiani et al. (2008) and van Ditmarsch et al. (2012) and Sequential Public Announcement Logic (SPAL) developed by Holliday (2018).

The learning process - L_{RKPTA} structures

Adam has put on his lab gown and is preparing to run experiments. Depending on the chosen course of action, he will run different experiments producing different results. Every time an experiment finishes he learns something new about the world. Subsequently, Adam updates his epistemic state based on his empirical observations and augments his knowledge. To model this dynamic process I will use an augmented version of L_{RKPT}, L_{RKPTA}, which includes announcements. The major additions take place in the structures of L_{RKPTA}. Most elements are carried over from L_{RKPT}, including the set of worlds W describing the state of affairs at any given moment, instantiated relations, and so on.

Time designations

The process starts in a world where Adam has complete ignorance and is thus unable to reject most epistemic possibilities. The one thing he knows is time. He can always locate himself in time using his trusted wristwatch. The watch is not a traditional clock and instead depicts integers. Fittingly, every world is assigned a number using the function

$Time : W \to \mathbb{Z}$. The clock accurately depicts the passage of time as described by $\prec$:

- $\forall v, u \in W : v \prec u \Rightarrow Time(u) = Time(v) + 1$

All the previously mentioned restrictions are also in place, including the uniqueness of future and past. As mentioned while discussing temporal frames, an important benefit of a deterministic framework is that the worlds can be organized into equivalence classes based on the timeline they belong to. Now a second classification into equivalence classes follows from the temporal designation of worlds $[t] = \{< w, v >\in W^2 : Time(w) = Time(v) = t\}$. The two equivalence relations, timelines $[w]$ and time designations $[t]$, organize the worlds in W into a grid. The intersection of a $[t]$ class and a $[w]$ is a single unique world. As a result, every world in W can uniquely be identified from its timeline and time designation as $[w]_t$.

Restrictions on epistemic relations

With his trusted clock, Adam can always tell at which point in time he is. To reflect that in the model, all epistemic relations must obey the following condition:

$$ERL \subseteq \{R \in \mathcal{P}(W^2) :< u, v >, < u, w >\in R \Rightarrow Time(v) = Time(w)\}$$

(temporal location)

Despite the apparent increase in complexity through the addition of conditions, the aim is to produce a simplified account of knowledge acquisition. To that end, I will adopt the controversial[7] step of adding transitivity and symmetry to the frame conditions of epistemic relations.

$$ERL \subseteq \{R \in \mathcal{P}(W^2) :< u, v >\in R \Rightarrow < u, v >\in R\} \quad \text{(symmetry)}$$

$$ERL \subseteq \{R \in \mathcal{P}(W^2) :< u, v >, < v, w >\in R \Rightarrow < u, w >\in R\} \quad \text{(transitivity)}$$

In the beginning, there is ignorance. A relation U that can do no better than knowledge of time is used to describe this oblivion. It is a universal clustering that aggregates all worlds with the same time notation. All worlds w with $Time(w) \leq 0$ are assigned this relation:

[7] Equivalence classes for epistemic relations guarantee positive ($Kp \to KKp$) and negative ($\neg Kp \to K\neg Kp$) introspection, resulting in knowledge of infinitely many introspective propositions.

$U := \{< u, w >\in W^2 : Time(u) = Time(w)\}$

$\forall w \in W : Time(w) \leq 0 \Rightarrow Ins(w) = U$

Adam can always tell apart the present from the future and the past. Yet before he starts gathering empirical facts, all alternative presents are epistemically possible. He is therefore unable to distinguish his timeline from the other possible ones. Yet every new announcement slowly discards timelines that contradict it.

Announcements

From moment 0 and onward, the empirical inquiry starts yielding results in the form of an announcement of a true proposition ϕ. Subsequently, the world changes to a new state to reflect the knowledge that ϕ was true a moment ago. Announcements are reliable and therefore only include true propositions. To depict announcements, the structure uses the function $AN : W \rightarrow FORM$, by which every world is assigned a formula expressing the last announcement. For moments before 0 where no announcements were taking place, no *a posteriori* knowledge is gained, thus any logical tautology can replace the announcement.

$AN([w]_t) = \top$, for all $t \in \mathbb{Z}$ with $t \leq 0$

Adam's epistemic relation in positive time can be defined recursively using the announcements.

First, some auxiliary definitions:

- $Next : W \rightarrow W$ a function that assigns to every world w the unique v such that $w \prec v$.
- $Prev : W \rightarrow W$ a function that assigns to every world w the unique v such that $v \prec w$.

Both *Next* and *Prev* are well defined given the frame conditions of no infimum, no supremum, unique future, and unique past.

The following can be trivially proven:

- $[w]_t \models A\phi \Leftrightarrow [w]_{t+1} \models \phi$
- $[w]_t \models B\phi \Leftrightarrow [w]_{t-1} \models \phi$
- $w \models A\phi \Leftrightarrow Next(w) \models \phi$
- $w \models B\phi \Leftrightarrow Prev(w) \models \phi$.

As a last preliminary, I define the epistemic update function $EU : \mathbb{N} \times FORM \rightarrow ERL$. The relation $EU(t, \phi)$ connects all the worlds whose timelines agree on ϕ at time t. It has the following definition:

$$EU(t, \phi) := \{< u, v > \in W \times W : Time(u) = Time(v) \wedge ([u]_t \models \phi \Leftrightarrow [v]_t \models \phi)\}$$

Finally, The instantiated relation on world w with $Time(w) > 0$ is given with the following recursive definition:

$$Ins(w) := Ins(Prev(w)) \cap EU(Time(w) - 1, AN(w)) \qquad \text{(epistemic updating)}$$

In simpler terms, after every announcement[8] the instantiated relation is modified to reflect the acquired knowledge that in the previous world, the announced proposition was true. Learning therefore consists of slowly pruning edges that contradict the empirical observations.

5.2 Epistemic updating - The power to know whether

The instantiated relation in every world is constructed from U following a series of announcements. This method of construction guarantees some useful properties. First, the construction through updating maintains temporal location, as the condition is satisfied in U, and is preserved after every update. Moreover, the product of every epistemic update is a more fine-grained partition of its predecessor. The proof by induction is straightforward since $A(t, \phi)$ is itself an equivalence relation, and the set of equivalence relations is closed under intersection. More importantly, though, all instantiated relations are time-independent.

[8] Holliday (2018) distinguishes between descriptive and hypothetical announcements. In epistemic updating, all announcements are descriptive as they generate the subsequent epistemic relation in some timeline. Instead of using hypothetical announcements, knowability will consist of reaching an alternative timeline with a different descriptive announcement.

Time independence is a property of some relations to remain the same when shifted forward or backward in time. This property is expressed by the following condition:

$$< [v]_t, [u]_k >\in R \Leftrightarrow \forall n \in \mathbb{Z} :< [v]_{t+n}, [u]_{k+n} >\in R \qquad \text{(time-independence)}$$

Construction through epistemic updating guarantees that all instantiated epistemic relations have this property.

Lemma 1. *For all t_0, $EU(t_0, \phi)$ is time-independent*

Proof.

Let $< [v]_t, [u]_k >\in EU(t_0, \phi)$.

By definition, $t = k$ and $[[v]_t]_{t_0} \models \phi \Leftrightarrow [[u]_k]_{t_0} \models \phi$

For every $n \in \mathbb{Z}$:

$t + n = k + n$

$[[v]_{t+n}]_{t_0} \models \phi \Leftrightarrow [[v]_t]_{t_0} \models \phi \Leftrightarrow [[u]_k]_{t_0} \models \phi \Leftrightarrow [[u]_{k+n}]_{t_0} \models \phi$

Thus $< [v]_{t+n}, [u]_{k+n} >\in EU(t_0, \phi)$. □

Theorem 3. *Instantiated relations constructed via epistemic updating are time-independent.*

Proof by induction.

Induction base: For $Time(w) \leq 0$, $Ins(w) = U$, and U is time-independent.

Induction step: Assume it is true for $Ins(w)$, we are going to prove time independence of $Ins(Next(w))$:

$Ins(Next(w)) = Ins(w) \cap EU(Time(w), AN(Next(w)))$

Let $< [v]_t, [u]_k >\in Ins(Next(w))$.

$\Rightarrow < [v]_t, [u]_k >\in Ins(w)$ and $< [v]_t, [u]_k >\in EU(Time(w), AN(Next(w)))$

Since both $Ins(w)$ (inductive hypothesis) and $EU(Time(w), AN(Next(w)))$ (above lemma) are time-independent, for every $n \in \mathbb{Z}$:

$< [v]_{t+n}, [u]_{k+n} >\in Ins(w)$ and $< [v]_{t+n}, [u]_{k+n} >\in EU(Time(w), AN(Next(w)))$

Therefore $< [v]_{t+n}, [u]_{k+n} >\in Ins(Next(w))$ and $Ins(Next(w))$ is time-independent.

□

Time independence and naive RKP

Time independence is crucial in maintaining factivity through naive RKP. For the knowability of ϕ in the original world o, naive RKP requires the existence of an epistemic

relation R whose horizon in o consists entirely of ϕ - worlds. Therefore $o \models K\phi[s(R/K)]$. Now assume that after several announcements we reach a world e with a new epistemic relation R' that can finally differentiate ϕ - worlds at time $Time(e)$. World v satisfies KA, which as was shown is not factive. If real evolution moved beyond the actual timeline, the evaluation of ϕ might be different in e. Nonetheless, R' can differentiate worlds at $Time(e)$ and thus, from time-independence, at $Time(o)$ as well.

From a logical perspective, the construction of instantiated relations via epistemic updating ensures that the satisfaction of the KA component entails the satisfaction of naive RKP. From a philosophical perspective, it describes how announcements give Adam the power to know whether ϕ is true regardless of the world he is in. A blind Adam would exclaim that if he had vision, he would know the color of the sky. Likewise, the color of the sky is knowable by an oblivious Adam if and only if, if he had the power to know whether it is blue or not, he would know it to be blue.

This power is imparted by the announcements. While announcements are always true propositions, in an epistemically indistinguishable timeline where the announced proposition is false, announcing its falsehood results in the same epistemic relation in the next world. This is evident from the definition of EU since:

$$EU(t, \phi) = EU(t, \neg\phi).$$

The event of an announcement splits the timelines into finer equivalence classes, each equipped with knowledge of whether the proposition is true. Looking at the sky results in the announcement "The sky is blue". Had it been red, a different proposition would have been announced. In both cases what matters is that looking at the sky is sufficient to know the truth regardless of what world is actual.

5.3 Announcement restrictions - Dynamic updates

Epistemic updates generate at each step the epistemic relation of tomorrow. For the recursive definition to be properly defined, however, the calculation of the next iteration $Ins(Next(w))$ must only use relations that have preceded it. Consider the example of announced proposition AEp at world w:

By recursion:

$$Ins(w) = Ins(Prev(w)) \cap EU(Time(w) - 1, AEp)$$

$$< u, v > \in EU(Time(w) - 1, AEp) \Rightarrow ([u]_{Time(w)-1} \models AEp \Leftrightarrow [v]_{Time(w)-1} \models AEp)$$

Imagine we want to decide whether $< w, v >$ belongs in $Ins(w)$. Assume $Time(w) = Time(v)$. The next step is evaluating whether $[w]_{Time(w)-1} \models AEp$.

$$[w]_{Time(w)-1} \models AEp \Leftrightarrow [w]_{Time(w)} \models Ep \Leftrightarrow w \models Ep$$
$$\Leftrightarrow \forall v \in W : Ins(w)(w, v) \Rightarrow v \models \phi$$

The evaluation ends abruptly when it requires $Ins(w)$, whose extension was being decided.

To avoid this vicious circle, announcements must be limited to propositions whose evaluation does not use instantiation relations of worlds with time designation greater or equal to $Time(w)$.

Announcement consistency

In theory, all announcements could be allowed, but then epistemic updating would not be a recursive definition, but a model condition reformulated as:

$$Ins(w) \subseteq (Ins(Prev(w)) \cap EU(Time(w) - 1, AN(w))) \quad \text{(announcement consistency)}$$

Given a structure and the instantiation function $Ins : W \rightarrow ERL$, announcement consistency could be verified. The result, however, would not be a dynamic system where every successor state is the product of previous states. The instantiating function *Ins* would have to be part of the frame. One could rightfully wonder how was the new relation generated. What factual truths account for the newly acquired knowledge? If announcements are supposed to model all empirical knowledge then any changes in the instantiated relation should be expressed as the product of one.

A pertinent example of this problem is a prophesy. Adam visits an oracle who reveals to him that tomorrow he will know the precise location of the holy grail. The prophecy can be accurate and Adam may come to know this secret the next day. The prophecy itself however does not tell him where the grail lies. A second epistemic update is required to depict this acquisition of knowledge. In an enumerable sequence of worlds, however, the announcement modeling each step cannot require a second intermediate infusion of knowledge.

In epistemic updating, the aim is to remove *Ins* from the frame and render it derivative

from facts of the world/timeline. Indeed, the sequence of announcements AN in a timeline $[w]$ uniquely decides the instantiated relation $Ins([w]_t)$ at any moment t. This move removes the possibility of paradoxes, like the above prophesy, caused by announcements of future knowledge.

Refining the announcement domain

To preserve the recursive definition, one option is to require a higher number of B than A operators to the left of epistemic operators. For this paper, I will simply reject the announcement of epistemic propositions altogether. Keep in mind that this move does not create a restrictive interpretation of knowability. As it will be shown, there are models where all propositions are knowable despite the limit to ontic announcements. The set of acceptable announcements will be referred to as $FORMA$:

$ONTIC := p \in P \mid \neg\phi \mid A\phi \mid B\phi$

$FORMA := p \in ONTIC \mid \top \mid \phi \wedge \psi$

Note that to create fine-grained relations, the epistemic update following an announcement of a conjunction must not announce the conjunction itself, but each of the conjuncts:

$EU(t, \top) := U$

$EU(t, p) := \{< u, v > \in W \times W : Time(u) = Time(v) \wedge ([u]_t \models p \Leftrightarrow [v]_t \models p)\}$

$EU(t, \phi \wedge \psi) := EU(t, \phi) \cap EU(t, \psi)$

5.4 Knowability as knowledge after announcement

With the expanded structures in place, the language can be augmented to describe possible announcements. The new operator $\langle\rangle$ will be interpreted as "there is a potential announcement". When the announcement ψ is specified, the operator will have the form $\langle\psi\rangle$, where $\psi \in FORMA$. The notion of potential once more requires an ability concept that will be expressed by real evolution. The semantic definitions are:

- $w \models \langle\psi\rangle\phi$ if and only if there exists a v with $RealEvo(w, v)$ such that:
  - $AN(v) = \psi$
  - $v \models \phi$
- $w \models \langle\rangle\phi$ if and only if there is a v with $RealEvo(w, v)$ such that $v \models \phi$

Real evolution can describe an ability concept as wide or narrow as the verification intuition needs it to be. For the model of empirical announcements, I will employ an epistemic ability concept:

- $RealEvo(w, v) \Leftrightarrow Ins(w)(w, prev(v))$

As an epistemic ability, real evolution only allows for transitions to timelines not yet excluded in the learning process. As mentioned while discussing ability concepts, this choice does not preserve truth but at least Adam will never contradict his existing knowledge in expecting a proposition to be knowable.

Note that by definition, operators $\langle\rangle$ and $\blacklozenge$ are equivalent[9]. I will use $\blacklozenge$ for general interpretations of real evolution, and $\langle\rangle$ for the particular epistemic interpretation $RealEvo(w, v) \Leftrightarrow Ins(w)(w, prev(v))$. Moreover, while other works in announcement logic use particular announcements $\langle\phi\rangle$ quite extensively, for simplicity I will focus exclusively on $\langle\rangle$.

Known after announcement

Using potential announcements, KAP can be improved further into knowledge acquisition of past truths after an announcement :

$$\exists K_1((E = K_1) \wedge \langle\rangle E(B\phi(K_1/E))) \qquad \text{(KAPA)}$$

The *de re* pass of the original epistemic relation can be omitted if the frame has the property of state awareness:

$$< w, v > \in Ins(w) \Rightarrow Ins(w) = Ins(v) \qquad \text{(state awareness)}$$

State awareness expresses that any worlds Adam considers possible through his actual epistemic state, also have the same epistemic state. It therefore guarantees that $\exists KE(E = K)$ is logically valid.

Theorem 4. $w \models \exists K_1((E = K_1) \wedge \langle\rangle E(B\phi(K_1/E))) \Leftrightarrow w \models \langle\rangle EB\phi$

[9] In traditional announcement logic, $w \models \langle\rangle\phi$ would be defined as $\exists\psi \in FORMA$ such that $w \models \langle\psi\rangle\phi$. Since every world has an announcement, $w \models \blacklozenge\phi$ guarantees the existence of a formula ψ. The other direction follows trivially from the second condition of $\langle\psi\rangle\phi$

Proof.

If $w \models \langle\rangle EB\phi(K/E)[s(Ins(w)/K)]$, then the real evolution hidden in $\langle\rangle$ leaps to a v with $Ins(w)(w, Prev(v))$, and thus $Ins(Prev(v)) = Ins(w)$. Moreover:

$w \models \langle\rangle EB\phi(K/E)[s(Ins(w)/K)]$

$\Leftrightarrow v \models EB\phi(K/E)[s(Ins(w)/K)]$

$\Leftrightarrow \forall v' \in Ins(v)_v : v' \models B\phi(K/E)[s(Ins(w)/K)]$

$\Leftrightarrow \forall v' \in Ins(v)_v : Prev(v') \models \phi(K/E)[s(Ins(w)/K)]$.

Since timelines are only removed and never introduced, $v' \in Ins(v)_v \Rightarrow Prev(v') \in Ins(Prev(v))_{Prev(v)}$

From state awareness:

$Prev(v') \in Ins(Prev(v))_{Prev(v)} \Rightarrow Ins(Prev(v')) = Ins(Prev(v)) = Ins(w)$.

Therefore:

$\forall v' \in Ins(v)_v : Prev(v') \models \phi(K/E)[s(Ins(w)/K)]$

$\Leftrightarrow \forall v' \in Ins(v)_v : Prev(v') \models \phi(K/E)[s(Ins(Prev(v'))/K)]$

$\Leftrightarrow \forall v' \in Ins(v)_v : Prev(v') \models \phi$

$\Leftrightarrow \forall v' \in Ins(v)_v : v' \models B\phi$

$\Leftrightarrow v \models EB\phi$

$\Leftrightarrow w \models \langle\rangle EB\phi$.

Therefore the *de re* pass of the original epistemic relation is no longer required. □

Under state awareness, KAPA can take the following form:

$$\langle\rangle EB\phi \qquad \text{(KAPA)}$$

The property of state awareness can be acquired in the construction of $Ins(w)$ by refining the equivalence relation further with:

$$Ins(w) := Ins(Prev(w)) \cap EU(Time(w) - 1, AN(w)) \cap SA(Time(w))$$

(state aware epistemic updating)

Relation $SA(t)$ is the equivalence relation of all co-temporal worlds whose timelines have the same announcement at time t:

$$SA(t) := \{< u, v > \in W \times W : Time(u) = Time(v) \wedge AN([u]_t) = AN([v]_t)\}$$

Finally, the construction of $Ins(w)$ creates an ordered sequence of instantiated relations for every timeline $[w]$:

$$Ins([w]_0) = U \supseteq Ins([w]_1) \supseteq Ins([w]_2) \supseteq Ins([w]_3) \supseteq Ins([w]_4) \supseteq ...$$

The combination of state awareness and epistemic real evolution makes it so $RealEvo(w, v)$ can only transition to worlds v whose timeline has the same epistemic past:

$$RealEvo(w, v) \Rightarrow Ins([w]_t) = Ins([v]_t) \text{ for all } t \leq Time(w)$$

(preserved epistemic history (PES))

With PES, knowability can look beyond the very next moment:

$$\langle\rangle_n EB_n\phi \text{ for some } n \in \mathbb{N} \qquad \text{(KAPA)}$$

where: $\langle\rangle_n := \langle\rangle\langle\rangle...\langle\rangle$ n times, and $B_n := BB...B$ n times.

Proposition ϕ is knowable if there is a potential series of announcements, after which it is known that ϕ was true in the original moment. It can be shown that KAPA is satisfiable for all formulas in $FORM$. In the next section, I present a class of models where KAPA is valid. It thus successfully overcomes the paradox of knowability without resorting to restrictions. With factivity defended by the naive RKP, the only problem left is potential omniscience.

5.5 Knowability in expansive models

KAPA relies on the existence of an alternate epistemically possible timeline, where a different proposition is announced in the following moment. If the existence of such a timeline is guaranteed then the validity of KAPA is ensured.

I will refer to the models that feature this property as expansive models. Formally, a model M is expansive (EM) if and only if:

- for all $w \in W$ and all formulas $\psi \in FORMA$, if $w \models \psi$ then there is a world v with $AN(v) = \psi$ and $Ins(w)(w, Prev(v))$
- for all $w \in W$, there is a $[v] \in Ins(w)_w$ that disagrees with $[w]$ about some $p \in P$ at some time t.

EM models express the confidence that any true ontic proposition can at any moment be discovered to be true following an empirical observation. A simple expansive model can be constructed by creating possible worlds for every possible valuation V. The next step is constructing all possible timelines of such worlds. Finally, identical timelines must be created for every possible sequence of announcements. At every moment t, timeline $[w]$, and $\phi \in FORMA$ true in $[w]_t$, there will be an epistemically possible timeline $[v]$ that agrees on all previously announced propositions but has $AN(t+1) = \phi$. The second condition is also satisfied as for every timeline, countless others agree with it in all propositions and announcements up to moment t, only to diverge at $t+1$.

The following section will prove that for any EM model, for all formulas ϕ in $FORM$ that do not include quantifiers or variables :

$$EM \models \phi \rightarrow \langle\rangle KB\phi \qquad \text{(KAPA verification)}$$

Preliminaries

To prove the validity of KAPA verification the following definitions are used:

- Given proposition $\phi \in FORM$, the left range $LRange$ of ϕ is the number of B appearances in ϕ. Inductively:
  - $LRange(p) := 0, p \in P$
  - $LRange(B\phi) := LRange(\phi) + 1$
  - the other cases are trivial.
- Given proposition $\phi \in FORM$, the right range $RRange$ of ϕ is the number of A and $\langle\rangle$ appearances in ϕ. Inductively:
  - $LRange(p) := 0, p \in P$
  - $LRange(A\phi) := LRange(\phi) + 1$
  - $LRange(\langle\rangle\phi) := LRange(\phi) + 1$
  - the other cases are trivial.

Furthermore the following lemmas are required:

Lemma 2 (E-equivalence)**.** $w \models E\phi \Leftrightarrow \forall v \in Ins(w)_w\colon v \models E\phi$

Proof.

Let $v \in Ins(w)_w$.

$w \models E\phi \Leftrightarrow \forall u(Ins(w)(w,u) \Rightarrow u \models \phi)$

From the symmetry and transitivity of equivalence relations:

$\forall u(Ins(w)(w,u) \Leftrightarrow Ins(w)(v,u))$.

From state awareness:

$Ins(w) = Ins(v)$.

Thus:

$\forall u(Ins(v)(v,u) \Rightarrow Ins(w)(v,u))$

$\Rightarrow \forall u(Ins(v)(v,u) \Rightarrow Ins(w)(w,u))$

$\Rightarrow \forall u(Ins(v)(v,u) \Rightarrow u \models \phi)$.

Therefore:

$v \models E\phi$.

The inverse is trivial from the reflexivity of $Ins(w)$. □

Note that E-equivalence entails $w \models E\phi \Leftrightarrow w \models EE\phi$, which is a form of positive introspection for instantiated relations. A similar proof can be given for negative introspection $w \models \neg E\phi \Leftrightarrow w \models E\neg E\phi$.

Lemma 3 (memory)**.** $w \models E\phi \Rightarrow Next(w) \models EB\phi$

Proof.

Let u such that $Ins(Next(w))(Next(w), u)$.

From time independence, $Ins(Next(w))(w, Prev(u))$.

By definition, $Ins(Next(w)) \subseteq Ins(w)$.

Thus:

$Ins(w)(w, Prev(u))$

$\Rightarrow Prev(u) \models \phi$

$\Rightarrow u \models B\phi$

$\Rightarrow Next(w) \models EB\phi$. □

Memory expresses the preservation of present knowledge in the future. Knowing that today it rained, Adam will know tomorrow that it rained yesterday. A perhaps unrealistic side-effect is that even ten years from now, he will remember the same fact about this

specific day in the past, a rare feat by human standards. Let us assume for simplicity that Adam keeps a rigorous history of his experiments.

Lemma 4 (hypothetical memory)**.** *If* $w \models E\phi$ *and* $RealEvo(w, v)$ *then* $v \models EB\phi$

Proof.

By epistemic evolution: $RealEvo(w, v) \Leftrightarrow Ins(w)(w, Prev(v))$.

From E-equivalence, $Prev(v) \models E\phi$.

From memory, $v \models EB\phi$. □

Hypothetical memory makes a similar claim to memory but about hypothetical futures. In no hypothetical future may Adam forget a truth he knows about today.

Lemma 5 (timeline pruning - TP)**.** *Let* $[v]_t$ *and* $[v]_d$ *two worlds belonging to the same timeline with* $t \leq d$*. For any timeline* $[u]$*, if* $[u] \in Ins([v]_d)_{[v]_d}$[10]*, then* $[u] \in Ins([v]_t)_{[v]_t}$.

Proof.

Let $[u] \in Ins([v]_d)_{[v]_d}$. Then from temporal location $[u]_d \in Ins([v]_d)_{[v]_d}$.

From time independence:

$\forall [u]$: $[u]_d \in Ins([v]_d)_{[v]_d} \Rightarrow [u]_t \in Ins([v]_d)_{[v]_t}$.

From PES:

$Ins([v]_d) \subseteq Ins([v]_t)$.

Thus:

$[u]_t \in Ins([v]_d)_{[v]_t} \Rightarrow [u]_t \in Ins([v]_t)_{[v]_t}$.

Therefore, $[u] \in Ins([v]_t)_{[v]_t}$. □

As the name suggests, TP describes how learning proceeds by removing timelines that contradict empirical announcements. If at any moment a timeline is epistemically possible, then so it was for every prior moment. New possibilities can never be introduced.

5.6 Informed worlds

The proof of KAPA verification will use a transition with real evolution to an informed world e. Given original world w and proposition ϕ, world e is informed of ϕ relative to w if and only if it fulfills the following criteria:

[10] For ease of use, while discussing timelines, I will use the notation $[v] \in R_w$ to mean that timeline $[v]$ is seen by w through R. Formally it would mean $R(w, [v]_{Time(w)})$.

- $Ins(w)(w, Prev(e))$ (real evolution)
- $AN(e) = Inform(\phi, w)$

Function $Inform : FORM \times W \rightarrow FORMA$ creates an announcement that can confer knowledge of the truth of ϕ in w. The announcement is constructed in the following way:

For every atomic proposition $p \in P$ present in ϕ:

$Fp := \overline{Ap} \wedge \overline{A_2 p} \wedge \overline{A_3 p} \wedge ... \wedge \overline{A_{RRange(\phi)} p}$

$Pp := \overline{Bp} \wedge \overline{B_2 p} \wedge \overline{B_3 p} \wedge ... \wedge \overline{B_{LRange(\phi)} p}$

Where:

$$\overline{\psi} := \begin{cases} \psi & \text{if } w \models \psi \\ \neg\psi & \text{if } w \not\models \psi \end{cases}$$

Finally $Inform(\phi, w) := \bigwedge(\overline{p} \wedge Fp \wedge Pp)$ for all $p \in P$ present in ϕ.

Lemma 6. *Let EM an expansive model. For every world w and formula ϕ true in it, there exists a world e that is informed of ϕ relative to w*

Proof. Let $\psi = Inform(\phi, w)$. Since ϕ has a finite number of propositional letters the conjunction is finite and thus $\psi \in FORMA$. Moreover, by construction, every conjunct of ψ is true in w, and therefore $w \models \psi$. Finally, from the definition of an *EM* model, the existence of world *e* is assured.

□

The announcement of $Inform(\phi, w)$ provides all relevant information to evaluate ϕ. World *e* thus becomes the coveted world where knowledge of ϕ is acquired. To prove the validity of the knowability thesis, I will prove the following stronger theorem:

Theorem 5 (Localised omniscience)**.** *Let w with $Time(w) = d$ the original world, ϕ a formula in FORM and e the informed world. For every timeline $[v]$ seen from e, and all times t and formulas $\psi \in FORM$ that only include propositional letters in ϕ such that $t + RRANGE(\psi) \leq d + RRANGE(\phi)$ and $t - LRANGE(\psi) \geq d - LRANGE(\phi)$:*

$$[v]_t \models \psi \Rightarrow \forall v \in Ins(e)_e : [v]_t \models \psi$$

Proof by induction on the complexity of ψ.

Induction base: Let ψ be a propositional letter p in ϕ. For any $t \in [d, d + RRANGE(\phi)]$, $\overline{A_{t-d}p}$ was part of the announcement in e. By the definition of $Ins(e)$ using EU, all timelines $[v]$ agree on the evaluation of $[v]_d \models \overline{A_{t-d}p}$. Without loss of generality assume $\overline{A_{t-d}p} = A_{t-d}p$. Then for all timelines $[v]$, $[v]_t \models p$. Therefore if $[v]_t \models p$ then $\forall v \in Ins(e)_e : [v]_t \models p$. Similarly for the case $\overline{A_{t-d}p} = A_{t-d}\neg p$.

Similar reasoning can be used for $t \in [d - LRANGE(\phi), d]$ since $\overline{B_{d-t}p}$ is part of the announcement in e.

Induction step: Let the inductive hypothesis hold for any proposition with a depth of its syntactic tree of k or less. Assume ψ has depth $k + 1$. It necessarily falls in one of the following cases:

- $\psi = \neg\theta$

  As a subtype of ψ, θ only includes propositional letters in ϕ. Moreover $RRange(\theta) = RRange(\psi)$ and $LRange(\theta) = LRange(\psi)$. Thus, if $[v]_t \models \psi$ fulfills the criteria of localised omniscience, so does $[v]_t \models \theta$.

  Assume $[v]_t \models \psi$.

  Let $u \in Ins(e)_e : [u]_t \not\models \psi$.

  $\Rightarrow [u]_t \models \neg\psi \Rightarrow [u]_t \models \neg\neg\theta \Rightarrow [u]_t \models \theta$

  By the inductive hypothesis on θ:

  $\forall z \in Ins(e)_e : [z]_t \models \theta$.

  Obviously, $[v] \in Ins(e)_e$, thus $[v]_t \models \theta$, contradicting the original assumption.

  Therefore $\forall u \in Ins(e)_e : [u]_t \models \psi$.

- $\psi = \theta \wedge \chi$

  If $[v]_t \models \psi$ fulfills the criteria of localised omniscience, so do $[v]_t \models \theta$ and $[v]_t \models \chi$.

  $[v]_t \models \psi \Leftrightarrow [v]_t \models \theta \wedge \chi$

  $\Leftrightarrow [v]_t \models \theta$ and $[v]_t \models \chi$.

  From the inductive hypothesis:

  $([v]_t \models \theta$ and $[v]_t \models \chi) \Rightarrow (\forall u \in Ins(e)_e : [u]_t \models \theta$ and $\forall u \in Ins(e)_e : [u]_t \models \chi)$

  $\Rightarrow \forall u \in Ins(e)_e : ([u]_t \models \theta$ and $[u]_t \models \chi)$

$\Rightarrow \forall u \in Ins(e)_e : [u]_t \models \theta \wedge \chi$

$\Rightarrow \forall u \in Ins(e)_e : [v]_t \models \psi.$

- $\psi = A\theta$

  Once again, θ as a subtype of ψ will only feature propositional letters that are present in ϕ. Moreover, $RRange(\psi) = RRange(\theta) + 1$ and $LRange(\psi) = LRange(\theta)$. Therefore if $t + RRANGE(\psi) \leq d + RRANGE(\phi)$, then $t + 1 + RRANGE(\theta) \leq d + RRANGE(\phi)$. As a result, $[v]_{t+1} \models \theta$ fulfills the inductive hypothesis.

  $[v]_t \models \psi \Leftrightarrow [v]_{t+1} \models \theta$

  $\Rightarrow \forall u \in Ins(e)_e : [u]_{t+1} \models \theta$

  $\Rightarrow \forall u \in Ins(e)_e : [u]_t \models A\theta$

  $\Rightarrow \forall u \in Ins(e)_e : [u]_t \models \psi$

- $\psi = B\theta$

  The proof follows identical reasoning as $\psi = A\theta$. The important observation is that $RRange(\psi) = RRange(\theta)$ and $LRange(\psi) = LRange(\theta) + 1$. Therefore if $t - LRANGE(\psi) \geq d - LRANGE(\phi)$, then $t - 1 - LRANGE(\theta) \geq d - LRANGE(\phi)$. As a result, $[v]_{t-1} \models \theta$ fulfills the inductive hypothesis.

- $\psi = E\theta$

  The proof is split into two cases:

  - $t \leq d + 1$

    $[v]_t \models \psi \Leftrightarrow [v]_t \models E\theta$

    Let $[u] \in Ins(e)_e$.

    Since $[v]$ is also in $Ins(e)_e$, it follows from state awareness that:

    $[u] \in Ins([v]_{d+1})_{[v]_{d+1}}$.

    From TP:

    $[u] \in Ins([v]_t)_{[v]_t}$.

    From E-equivalence:

    $[u]_t \models E\theta$.

    Therefore, $\forall u \in Ins(e)_e : [u]_t \models E\theta$.

– $t \geq d + 1$

$[v]_t \models \psi \Leftrightarrow [v]_t \models E\theta$

From the reflexivity of equivalence relations:

$\Rightarrow [v]_t \models \theta$.

From the inductive hypothesis on $[v]_t \models \theta$:

$\forall v \in Ins(e)_e : [v]_t \models \theta$.

Let $[u] \in Ins(e)_e$.

Let $[z] \in Ins([u]_t)_{[u]_t}$.

From TP: $[z] \in Ins([u]_{d+1})_{[u]_{d+1}}$.

From state awareness: $[z] \in Ins(e)_e$.

thus $[z]_t \models \theta$ and $[u]_t \models E\theta$.

Finally: $\forall u \in Ins(e)_e : [u]_t \models E\theta$.

- $\psi = \langle\rangle\theta$

  Once more there are two cases:

  – $t \leq d + 1$

  $[v]_t \models \langle\rangle\theta \Leftrightarrow \exists z : RealEvo([v]_t, z) \wedge z \models \theta$

  $\Leftrightarrow \exists z : Ins([v]_t)([v]_t, Prev(z)) \wedge z \models \theta$

  Let $[u] \in Ins(e)_e$.

  Since $[v]$ is also in $Ins(e)_e$, it follows from state awareness that:

  $[u] \in Ins([v]_{d+1})_{[v]_{d+1}}$.

  From TP:

  $[u] \in Ins([v]_t)_{[v]_t}$.

  From state awareness and the symmetry and transitivity of equivalence relations:

  $Ins([u]_t)([u]_t, Prev(z))$ and thus $RealEvo([u]_t, z)$.

  Therefore, $[u]_t \models \langle\rangle\theta$.

  – $t \geq d + 1$

  $[v]_t \models \langle\rangle\theta \Leftrightarrow \exists z : Ins([v]_t)([v]_t, Prev(z)) \wedge z \models \theta$

$\Rightarrow [z] \in Ins([v]_t)_{[v]_t}$

From TP:

$\Rightarrow [z] \in Ins([v]_{d+1})_{[v]_{d+1}} \Rightarrow [z] \in Ins(e)_e$

Moreover, $Time(z) = t + 1$, $RRange(\psi) = RRange(\theta) + 1$ and $LRange(\psi) = LRange(\theta)$. Therefore if $t + RRANGE(\psi) \leq d + RRANGE(\phi)$, then $t + 1 + RRANGE(\theta) \leq d + RRANGE(\phi)$. As a result, $z \models \theta$ fulfills the inductive hypothesis.

$\forall u \in Ins(e)_e : [u]_{t+1} \models \theta$

Lastly, $RealEvo([u]_t, [u]_{t+1})$ from the reflexivity of $Ins([u]_t)$.

Therefore, $\forall u \in Ins(e)_e : [u]_t \models \langle\rangle\theta$.

This concludes all the cases for ψ. Thus by induction localised omniscience is true.

□

Theorem 6 (KAPA verification)**.** *For any EM model, for all formulas ϕ in FORM that do not include quantifiers or variables :*

$$EM \models \phi \rightarrow \langle\rangle KB\phi$$

Proof.

A particular case for localized omniscience is $Prev(e) \models \phi$. It fulfills the criteria for localized omniscience and therefore:

$\forall u \in Ins(e)_e : [u]_d \models \phi$

$\Rightarrow \forall u \in Ins(e)_e : [u]_{d+1} \models B\phi$

and finally:

$e \models EB\phi$.

In an *EM* model, the informed world e can be constructed for all worlds w and formulas $\phi \in FORM$ without quantifiers or variables. Thus:

$$\models \phi \rightarrow \langle\rangle KB\phi \qquad \text{(KAPA verification)}$$ □

5.7 Potential omniscience

The problem of omniscience has been a thorn in the side of verification ever since the Fitch paradox. KAPA allows for the knowability of Fitch sentences as $EB(p \wedge \neg Ep)$ is satisfiable. Omniscience though still lurks around the corner. It is theoretically possible that in some future world v reachable through real evolution, the instantiated relation $Ins(v)$ can

differentiate all propositions. In that case, for all $p \in P : v \models p \rightarrow Ep$ and thus, from time independence, in the original world $w \models p \rightarrow \langle\rangle_n EB_n p$. That result would be potential omniscience. As Heylen (2020b) argues, Fuhrmann (2014)'s potential knowledge suffers from this very problem.

Understanding why potential omniscience is untenable is crucial for any effort to avoid it. In the possible knowledge interpretation of knowability, actual omniscience is factually wrong, hence why the Fitch paradox is catastrophic. On the contrary, potential omniscience seems at first glance merely implausible. In that case, the model could allow for states where the identity relation is instantiated, claiming it to be a miraculous occurrence. It would be no different than a Markov chain model of a coin toss allowing the perpetual appearance of Heads results. It is a distinct possibility but beneath the notice of a pragmatic analysis.

There are, however, reasons to believe that a state of omniscience is humanly impossible. The main argument is that a finite agent cannot know infinitely many ontic truths, even though infinite truths can be knowable by him. Assume that indeed in the actual timeline all truths will eventually be known. That means that for every truth $\phi \in FORM$ there is a moment t after which it is known. It does not follow, however, that there is a moment t after which all truths are known. The latter would require a finite agent to become infinite, while the former merely to continuously grow his knowledge while remaining finite. An interpretation of knowability should safeguard the first possibility while abolishing the second.

There are multiple ways to force the model to disallow potential omniscience. One solution is to reject timelines with worlds where omniscience is true. Another is to allow them but limit real evolution to avoid them. Even then, such proposals risk sacrificing the verification thesis. The more elegant solution is to prevent its emergence by the construction of $Ins(w)$. The dynamic model of empirical announcements achieves just that. Announcements consist entirely of ontic truths. Any conjunction of ontic truths, no matter how large, remains finite and is thus unable to confer knowledge of an infinitude of other propositions.

In models of only a handful of timelines, potential omniscience is inevitable. In larger models, like the expansive models, potential omniscience is impossible. In any world, the new $Ins(w)$ will be able to differentiate worlds based on the announcement, but there will always be much more that Adam does not yet know.

5.8 RKP verification in epistemic updating

The last step in defining knowability via epistemic updating is to organize all the conditions into one coherent operator $\langle RKP \rangle$. The language can shed the now-outmoded variables and quantifiers over epistemic relations for the following laconic interpretation of the verification thesis

$$\phi \leftrightarrow \langle RKP \rangle \phi \qquad \text{(RKP verification)}$$

The semantic interpretation of $\langle RKP \rangle$ is as follows:

$w \models \langle RKP \rangle \phi$ if and only if there is a world $v \in W$ such that:

- $Prev(v) \in Ins(w)_w$
- $v \models EB\phi$
- for all z in $Ins(v)_w$: $z \models \phi$.

The first two conditions reflect KAPA knowability. As demonstrated by localized omniscience, for *EM* models there is always an informed world e that satisfies them. The final condition reflects the naive RKP and demands that the new epistemic relation would grant knowledge to the agent at w, where he to hypothetically poses it. This condition renders the interpretation factive since $w \in Ins(v)_w$. If the informed world e also satisfies the third condition, then RKP verification is also valid in *EM* models while being factive and avoiding potential omniscience. However, proof of the latter is still pending.

In its stead, I will present a formal example to demonstrate the claim of RKP verification.

Formal example

The example will focus on a small model M consisting of four timelines, $[w]$, $[v]$, $[u]$ and $[z]$. The set of atomic propositions P consists of letters p and q. The following graph (Figure 2) depicts the model at times $t = 0$, $t = 1$, and $t = 2$.

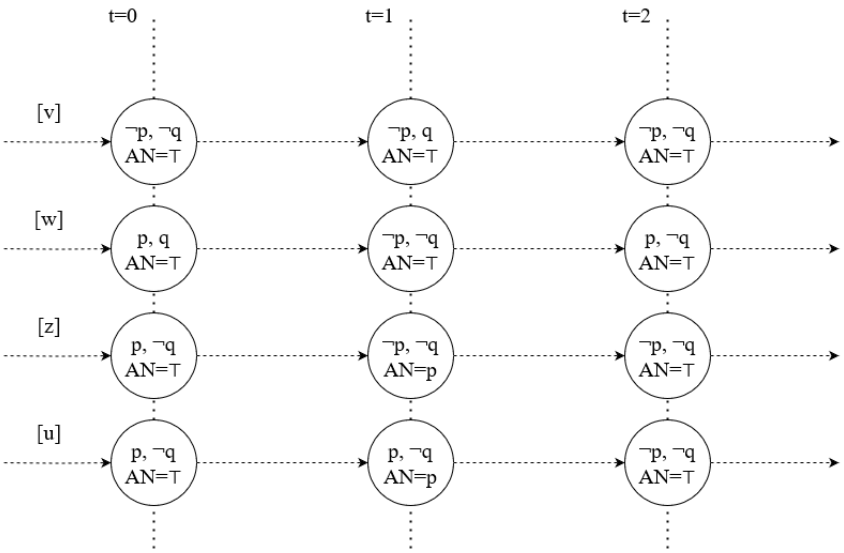


Figure 2: The example model between times $t = 0$ and $t = 2$. Each world is assigned a valuation for p, q, and an announcement AN. The horizontal lines describe the timelines while the vertical the time designations.

Note that the model does not directly include the *Ins* function, as instantiated relations are derived from announcements. At time $t = 0$, all worlds have the same instantiated relation U (Figure 3):

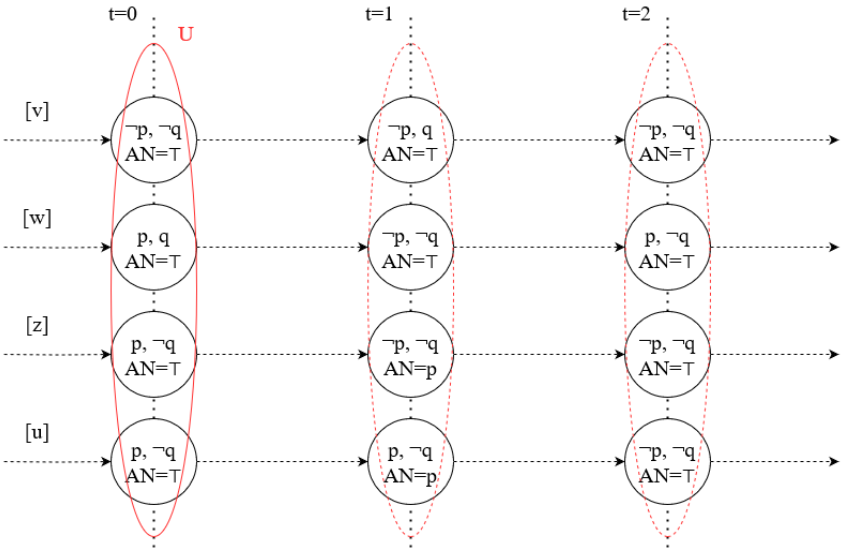


Figure 3: Relation U is the instantiated relation for all worlds at time $t = 0$. Since epistemic relations are equivalences, they can be described by delineating their equivalence classes, with every circle describing a cluster. The full line depicts the set of worlds where the relation is instantiated

In this example, we will test the RKP verification thesis on world $[w]_0$ and proposition

p. From the definition of $w \models \langle RKP \rangle \phi$, we are looking for a $w' \in W$ that fulfills the following criteria:

1. $Prev(w') \in Ins([w]_0)_{[w]_0}$
2. $w' \models EBp$
3. for all z in $Ins(w')_{[w]_0}$: $z \models p$

The first step is looking for worlds that are reachable from $[w]_0$ via real evolution, those being the epistemically possible futures. Given $Ins([w]_0) = U$, the following worlds are candidates for w' (Figure 4):

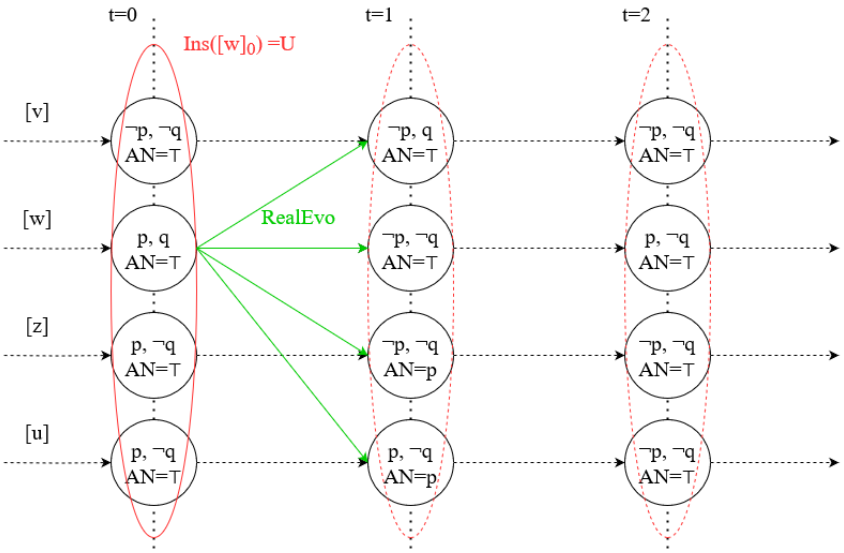


Figure 4: Accessible worlds from $[w]_0$ via real evolution

The next step is evaluating EBp in those worlds. For that, we need their instantiated relations. Those are calculated using state-aware epistemic updating:

$$Ins(w) := Ins(Prev(w)) \cap EU(Time(w) - 1, AN(w)) \cap SA(Time(w)).$$

The new relations will aggregate worlds that agree on the announcement at $t = 1$, and the announced proposition at $t = 0$. The result is the following two relations (Figures 5 and 6):

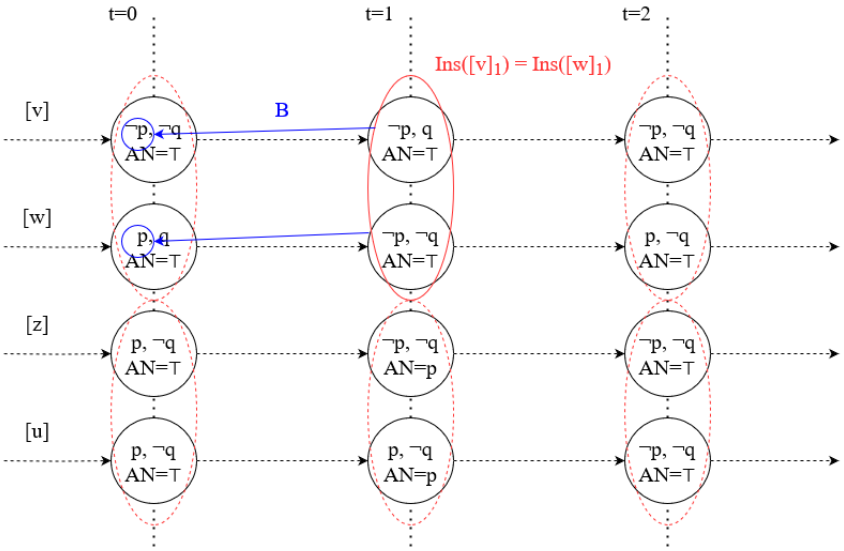


Figure 5: Instantiated relation for $[w]_1$ and $[v]_1$ and the evaluation of Bp

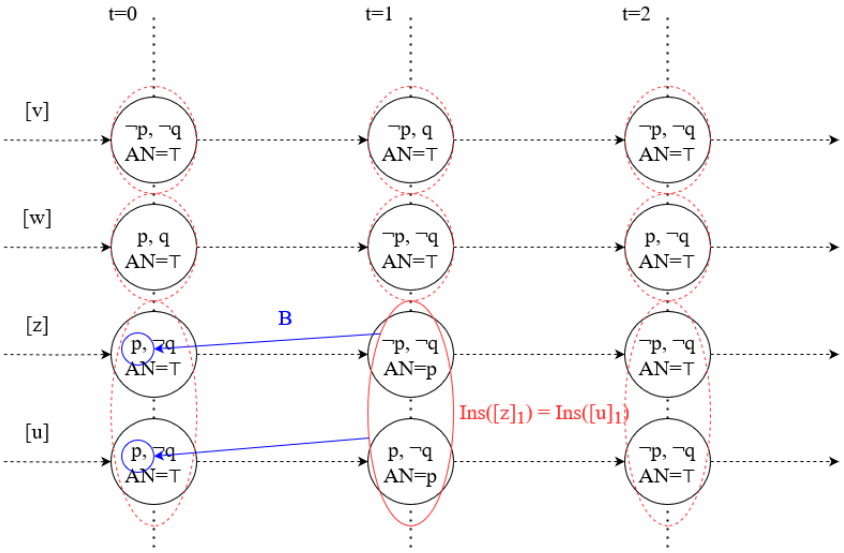


Figure 6: Instantiated relation for $[z]_1$ and $[u]_1$ and the evaluation of Bp

The second criterion is only fulfilled in worlds $[z]_1$ and $[u]_1$ where the announcement was p. Therefore, these two worlds satisfy KAPA verification. For RKP verification, however, a third condition must be checked. As per naive RKP, the new relation $Ins([z]_1)$ should hypothetically grant knowledge to the agent at $[w]_0$ if it had been instantiated there. To that end, p must be true in every world in $Ins([z]_1)_{[w]_0}$ (Figure 7):

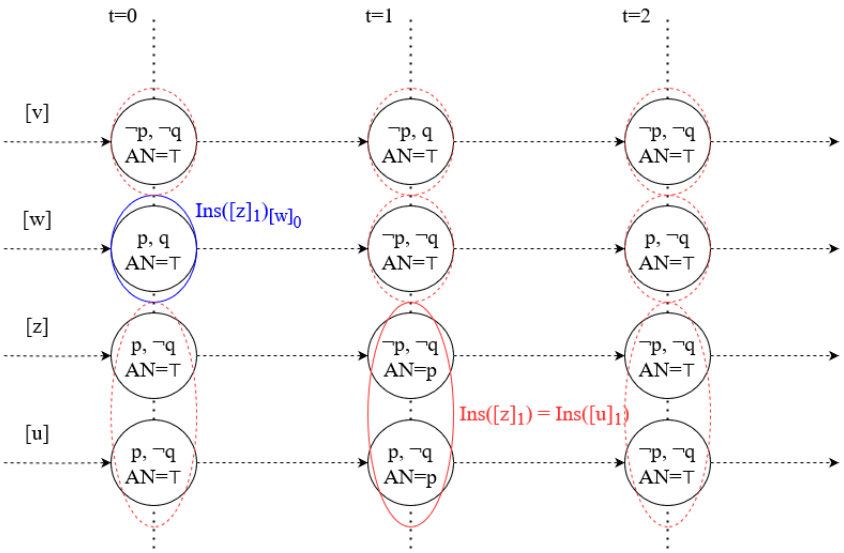


Figure 7: Checking naive RKP for $Ins([z]_1)$

This is indeed the case as the only timeline in $Ins([z]_1)_{[w]_0}$ is $[w]$ and $[w]_0 \models p$. Thus p is RKP knowable in $[w]_0$ through announcement $AN = p$ at $[z]_1$. Moreover, the same real evolution renders the Fitch sentence $p \wedge \neg Ep$ knowable. After all, $Inform(p \wedge \neg Ep, [w]_0) = p = AN([z]_1)$.

The importance of the last condition becomes evident when evaluating the knowability of $\neg q$. The same announcement $AN = p$ in $[z]_1$ happens to render $EB\neg q$ true in that world. From KAPA knowability, $\neg q$ would be knowable at $[w]_0$ even though it is false, thus breaking factivity. The third condition blocks the knowability of falsehoods, as $\neg q$ is not true in $Ins([z]_1)_{[w]_0}$ and thus would not have been known by the agent if he had the same instantiated relation as in world $[z]_1$ (Figure 8).

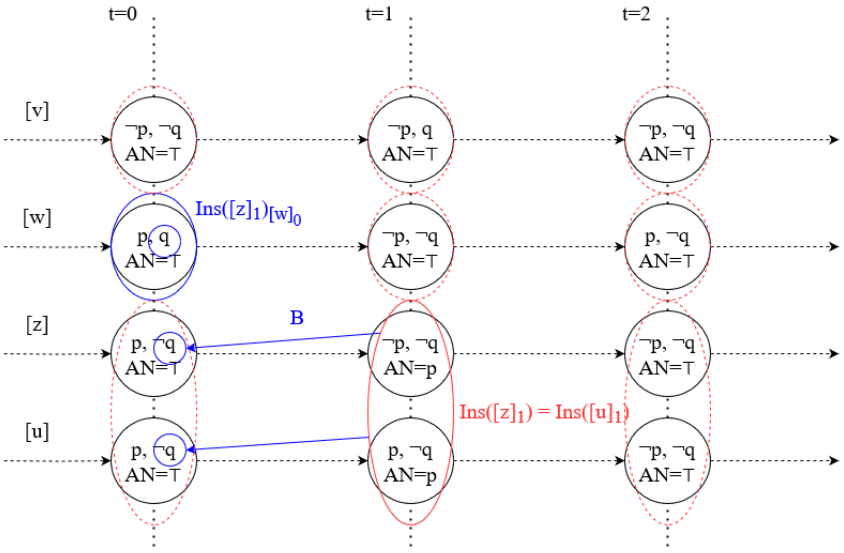


Figure 8: $\neg q$ is KAPA knowable but not RKP knowable

5.9 Final remarks on epistemic updating - Comparisons with APAL and SPAL

Epistemic updating relies on structures depicting possible worlds organized in timelines. Depending on the model, RKP can be true or false, potential omniscience can be avoidable or inescapable. Should the frame consist of only one timeline, then Adam already knowns everything about his world. In an expansive model that includes all possible timelines, KAPA is valid and omniscience is avoidable.

This dependence on the model for the evaluation of knowability is one of the many differences epistemic updating has with APAL and SPAL. In APAL developed by Balbiani et al. (2008) and later van Ditmarsch et al. (2012), the operator $\langle\psi\rangle$ is interpreted differently than epistemic updating. Instead of searching the worlds present in the model M for one with a fitting announcement, (APAL) creates a new hypothetical model $M|\psi$ where all worlds that disagree on ψ are removed. The verification principle is then shown to be trivially valid for "successful" formulas, those being all ϕ that are true in $M|\phi$, regardless of model M.

SPAL developed by Holliday (2018) also creates a valid interpretation of verificationism. Similar to epistemic updating, SPAL uses a sequence of epistemic states depicting knowledge at different times. This epistemic sequence, however, is not tied to a timeline. In the same vein as APAL, the operator $\langle\psi\rangle$ does not search existing timelines for one with the desired epistemic sequence. Instead, SPAL creates a new hypothetical sequence ad hoc, one that is identical in present and past relations, but where all future relations can differentiate worlds based on ψ. Once more the validity of verificationism is trivial as announcing any formula renders it knowable.

In epistemic updating, RKP verification is not a logical validity. Its truth can be assured by employing an expansive model. The difference in the approach is intentional, as I do not believe a defense of verification against the paradox of knowability should prove its validity, but merely its consistency. The Fitch paradox itself used the possible knowledge interpretation, which similarly to RKP is not valid. In philosophical debates, logic can shed light on controversies by dissolving the ambiguities of natural language and revealing salient presuppositions. It is, however, important for logic to remain neutral and not overstep its reach, lest it too express concealed intuitions. Seemingly innocuous abstractions during framework crafting can axiomatically infuse the system with contentious side effects. This very paper is based on one such observation. By keeping the evaluating

world intact and only changing epistemic relations, Holliday (2018), adopting the strategy from Fuhrmann (2014), can ensure factivity. Yet how can alternate sequences of epistemic states be expressed in the facts of a single world.

Finally, a last distinction of RKP from APAL and SPAL is the absence of restrictions. Restricting the domain of knowable formulas was one of the main tactics for overcoming the Fitch paradox. Limiting formulas to ontic propositions ignores the paradox as the Fitch sentence $p \wedge \neg Kp$ is an epistemic proposition. A similar restrain is employed by epistemic updating, but only for announcements. As it was proven, all formulas are knowable despite this use of limited announcements. On the contrary, both APAL and SPAL are restrictive interpretations of knowability. APAL is limited to successful formulas while SPAL only allows formulas without futuristic operators.

6 Conclusion

Of the many reinterpretations of knowability, Fuhrmann (2014)'s potential knowledge seems to come the closest to overcoming the Fitch paradox without sacrificing factivity albeit with the danger of regressing to potential omniscience. Unfortunately, the rigid world structure he employs is incommensurable with the epistemic changes he demands.

In this paper, I have attempted to translate the evolution of epistemic relations into a transformation of worlds by developing the RKP framework. The key element is the notion of real evolution which describes an ability concept for accessing non-actual worlds. Using the observation that a different agent can know the Fitch sentence, I defined different interpretations of knowability each with their advantages and disadvantages. The naive RKP preserves factivity but does little to express a gain in knowledge. On the contrary, the KA component describes a transition to a world of knowledge, but can also result in knowledge of hitherto falsehoods.

For a more nuanced analysis, a temporal structure was added to the interpretation. The set of possible worlds is now organized in timelines and real evolution can opt to respect the passage of time or transcend it. Timelines enable refined interpretations of knowability, from actual knowability which states that all truths will be known in the future, to epistemic knowability which allows for non-actual albeit epistemically possible futures. Moreover, with the aid of temporal operators, the KA thesis was improved to KAP to describe knowledge of past truths.

Finally, I applied the framework of RKP to describe a simple empirical learning process. The model of epistemic updating depicts the acquisition of *a-posteriori* truths through announcements that deterministically alter the agent's epistemic state. A natural continuation of KAP, KAPA envisions real evolution as the existence of an announcement that can lead to knowledge. Using simplifications such as an epistemic account of real evolution and epistemic relations that respect equivalence, I demonstrated that even if the announced truths are limited to ontic propositions, the verification thesis is valid in sufficiently expansive models.

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Abstract

The verification thesis claims that all truths are knowable, rejecting the notion of hidden truths, albeit allowing hitherto unknown facts. The Fitch paradox of knowability poses a devastating critique of verificationism by seemingly deriving from it omniscience, the state of affairs where all truths are already known. Different defenses have been raised against the paradox, among them the denial that knowability should be understood as possible knowledge. Various reinterpretations have been offered to provide an alternative formalization of knowability that is immune to the paradox.

This paper uses the potential knowledge interpretation developed by Fuhrmann (2014) as a point of departure for a new solution. Potential knowledge understands knowability as an evolution of the agent's epistemic state, described by a relation between worlds, following new evidence. However, given the supervenience of epistemic states on natural facts, any evolution of epistemic states must also be represented in changes in the actual world, a point that Fuhrmann (2014) overlooks. To that end, I developed a new interpretation, knowability as real knowledge potential (RKP), which assigns an epistemic relation to every world, portraying the dependence of the agent's epistemic state on natural truths. Epistemic evolution is replaced by real evolution, a possibility operator that traverses the possible worlds in search of a different epistemic state. Using this framework I define various interpretations of knowability with different advantages and disadvantages.

Moreover, I add a time dimension to the definition of knowability to capture the verification intuition as knowledge in some future. Temporal structures allow for a nuanced analysis that can account for knowable truths that will never be known or truths that can only be known about the past. This approach also reveals the difficulties of creating factive accounts of knowability that reject the knowability of falsehoods without being limited to the actual sequence of events.

In the last part of this paper, I apply the model of real knowledge potential to describe a dynamic empirical learning process. Inspired by dynamic approaches in the literature, I model the acquisition of empirical facts as announcements. Every world is outfitted with an announced truth and learning consists of a path along possible worlds and the accompanying gradual aggregation of knowledge with every new revelation. I then present the results of the framework regarding validity, factivity, and potential omniscience and compare them to other dynamic accounts of knowability.

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